Since A and B are mutually exclusive event therefore probability = $P(A\cup B)=P(A)+P(B)$

$$P(A\cup B)=P(A)+P(B)$$

$$P(A\cup B)=\frac{n(A)}{n(S)}+\frac{n(B)}{n(S)}$$

$$P(A\cup B)=\frac{10}{60}+\frac{30}{60}$$

$$P(A\cup B)=\frac{40}{60}$$

$$P(A\cup B)=\frac{2}{3}$$

2019

61: How many words can be formed from the letters of "PLAN" using all letters when no letter is to be repeated?

Sol: Given Word **PLAN**

Numbers of letters = n=4

Using letters = r = 4

Total number of words = $^{n}P_{r}={}^{4}P_{4}$

=24

2021

62. Write $\frac{52.51.50.49}{4.3.2.1}$ in the factorial form.

Sol: $\frac{52.51.50.49}{4.3.2.1}=\frac{52.51.50.49}{4!}$

Multiply and divide by 48!

$$=\frac{52.51.50.49.48!}{4!\ \ 48!}$$

$$=\frac{52!}{4!\ \ 48!}$$

63- Evaluate $^{9}P_{8}$

Given $^{9}P_{8}$

We know that

$$^{n}P_{r}=\frac{n!}{(n-r)!}$$

$$^{9}P_{8}=\frac{9!}{(9-8)!}$$

$$=\frac{9!}{1!}$$

$$=\frac{9.8.7.6.5.4.3.2.1}{1}=362880$$

64. How many arrangements of the letters of the word ATTACKED" can be made if each arrangement begins with C and ends with K .

Sol: ATTACKED

Let each arrangement begins with c and ends with k, then the words of the forms

$C \times \times \times \times \times \times K$

Total no. of letters other than c and k = 6

A is repeated = 2 times

T is repeated = 2 times

D is repeated = 1 times

E is repeated = 1 times

Total number of arrangements = $\binom{6}{2,2,1,1}$

$= \frac{6!}{2!.2!1!.!1!}$

$= \frac{6.5.4.3.2.1}{2.1.2.1.1.1} = 180$

65- A dice is thrown twice, what is the probability that the sum of the numbers of dots shown 3 or 11

Sol: When two dices are rolled then

$$S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

$\Rightarrow n(S) = (6)^2 = 36$

Let A be the event that sum of dots are 3

$A = \{(1,2),(2,1)\}$

$n(A) = 2$

$P(A) = \frac{n(A)}{n(S)} = \frac{2}{36}$

Let B be the event that sum of dots are 11

$B = \{(5,6),(6,5)\}$

$n(B) = 2$

$P(B) = \frac{n(B)}{n(S)} = \frac{2}{36}$

Since A and B are disjoint events

So,

$P(A \cup B) = P(A) + P(B)$

$P(3 or 11) = \frac{2}{36} + \frac{2}{36} = \frac{2+2}{36} = \frac{4}{36} = \frac{1}{9}$

66- Evaluate ${}^{16}P_4$

Sol: Given ${}^{16}P_4$

We know that

$${}^{n}P_r = \frac{n!}{(n-r)!}$$

$${}^{16}P_4 = \frac{16!}{(16-4)!}$$

$$= \frac{16!}{12!}$$

$$= \frac{16.15.14.13.12!}{12!}$$

$$= 16.15.14.13 = 43680$$

67- In how many ways can a necklace of 8 beads of different colours be made?

Sol: Now of beads = n = 8

Required no. of arrangements $= \frac{1}{2}(n-1)!$

$$= \frac{1}{2}(8-1)!$$

$$= \frac{1}{2}(7)!$$

$$= \frac{1}{2}(5040) = 2520$$

LONG QUESTION'S OF CHAPTER-7 IN ALL PUNJAB BOARDS 2011-2021

1. Find the values of n and r, when ${}^{n}C_r = 35$ and ${}^{n}P_r = 210$
2. Find the values of n and r when ${}^{n}C_r = 35$ and ${}^{n}P_r = 210$
3. How many numbers greater than 1000,000 can be formed from the Digits 0, 2, 2, 2, 3 ,4 ,4
4. A natural number is chosen out of the first fifty natural numbers. What is the probability that the chosen number is a multiple of 3 or of 5.
5. Prove that ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$
6. Prove that ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$
7. Two dice are thrown. What is the probability that the sum of the number of dots appearing on them is 4 or 6?
8. Find the values of n and r, when ${}^{n}C_r = 35$ and ${}^{n}P_r = 210$
9. Prove that ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$
10. Find the values of n and r, when ${}^{n}C_r = 35$ and ${}^{n}P_r = 210$
11. Prove that ${}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^{n}C_r$
12. Prove that ${}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^{n}C_r$

OBJECTIVE MCQ'S OF CHAPTER-8 IN ALL PUNJAB BOARDS 2011-2021

Topic I: Mathematical Induction:

1. $\forall n \in N$, $5^n - 2^n$ is dividable by: **(3 times)**
(A) 2 (B) 3 (C) 4 (D) 5

2. The method of induction was devised by: **(2 times)**
(A) Laplace (B) Aristotelian (C) Francesco Mourolico (D) Euclid

3. The inequality $4^n > 3^n + 2^{n-1}$ is true for integral values of n if **(2 times)**
(A) $n > 0$ (B) $n \geq 1$ (C) $n \geq 2$ (D) $n \leq 2$

4. If n is positive integer then one factor $x^n - y^n$ is: **(4 times)**
(A) $x + y$ (B) $x - y$ (C) $x^{-1} + y^{-2}$ (D) $(1 - x)^2$

5. $1 + 2 + 3 + 4 +$ ---------------- $+ n$ equals **(2 times)**
(A) $\frac{n(n-1)}{2}$ (B) $\frac{n(n+1)}{2}$ (C) $\frac{n+1}{2}$ (D) $\frac{n(n+1)^2}{2}$

6. If n is any +ve integer then $4n > 3n + 4$ is true for all:- **(2 times)**
(A) $n > 0$ (B) $n \geq 2$ (C) $n < 2$ (D) $n \leq 2$

7. If n is any positive integer, then $2^n > 2(n + 1)$ is true for all: **(2 times)**
(A) $n \leq 3$ (B) $3 < 3$ (C) $n \geq 3$ (D) $n > 3$

8. If $n^2 > n + 3$ then it is true for:
(A) $n = 0$ (B) $n < 1$ (C) $n \geq 2$ (D) $n \geq 3$

9. The inequality $n! > 2^n + 1$ is valid if:
(A) $n < 4$ (B) $n \geq 4$ (C) $n \leq 3$ (D) $n = 3$

10. If n is positive integer, then $n^2 > n + 3$ is true when: **(2 times)**
(A) $n \geq 3$ (B) $n \geq 2$ (C) $n \geq 1$ (D) $n \leq 3$

Topic II: Binomial Theorem with Positive Integers:

11. $\binom{n}{r} a^{n-r}b^r$ is __________ term of $(a+b)^n$? **(2 times)**
(A) (r+1)th (B) (r+2)th (C) (r+3)th (D) r th

12. The middle terms of $(x + y)^{23}$ are: **(6 times)**
(A) 10th and 11th (B) 11th and 12th (C) 12th and 13th (D) 13th and 14th

13. The second term in the expansion of $(1 + 2x)^{1/2}$ is: **(1 time)**
(A) x (B) 2x (C) 3x (D) 4x

14. The no. of term independent of x in the expansion of $\left(\frac{x}{2} + \frac{2}{x^2}\right)^{12}$ is:-
(A) 5 (B) 6 (C) 7 (D) 8

15. General term of the expansion $(a + b)^n$ is:- **(5 times)**
(A) $\binom{n}{r} a^{n-r}b^{n-r}$ (B) $\binom{n}{r} a^{n-r} b^r$ (C) $\binom{n}{r} a^n b^{n-r}$ (D) $\binom{n}{r} a^n b^r$

16. If n is even then the middle term in the expansion of $(a + b)^n$ is: **(3 times)**
(A) $\left(\frac{n}{2}\right)$th term (B) $\left(\frac{n+2}{2}\right)$th term (C) $\left(\frac{n+1}{2}\right)$th term (D) $\left(\frac{n+3}{2}\right)$th term

17. Number of terms in the expansion of $(a + b)^{11}$ is: **(2 times)**
(A) 11 (B) 12 (C) 10 (D) 6

18. In the expansion of $(a + x)^n$ (if n is even) middle term is :- **(1 time)**
(A) $\left(\frac{n}{2} + 1\right)$th (B) $\frac{n+1}{2}$ th (C) $\frac{n}{2}$ th (D) $\frac{n-1}{2}$ th

19. The coficient of second term in the expansion of $(a + b)^{20}$ is: **(6 times)**
(A) 18 (B) 19 (C) 20 (D) 21

20. In the expansion of $(x - y)^n$ the terms are alternative +ve and ....
(A) negative (B) undefined (C) absurd (D) None

21. The number of terms in the binomial expansion of $(a + b)^{15}$ are: (4 times)
(A) 15 (B) 14 (C) 16 (D) 13

22. General term of expansion $(a + x)^n$ is: (2 times)
(A) $\binom{n+1}{r}a^{n-r}x^r$ (B) $\binom{n}{r-1}a^{n-r}x^r$ (C) $\binom{n}{r+1}a^r x^{n-r}$ (D) $\binom{n}{r}a^{n-r}x^r$

23. Coefficient of 8th term in the expansion of $(a+ b)^{10}$ is:
(A) ${}^{10}C_7$ (B) ${}^{10}P_8$ (C) ${}^{8}C_{10}$ (D) ${}^{8}P_{10}$

24. Sum of binomial coefficients is:
(A) n (B) 2^n (C) 2n (D) n^2

25. In the expansion of $(a+x)^{2n}$, number terms is equal to:
(A) n + 1 (B) 2n + 1 (C) n + 2 (D) 2n + 2

26. An algebraic expression consisting of two terms is called:
(A) Monomial (B) Trinomial (C) Polynomial (D) Binomial

27. Number of terms in the expansion of $(a+b)^n$ is: (2 times)
(A) n (B) n + 1 (C) n - 1 (D) n - 2

28. The sum of exponents of a and x in every term of expansion of $(a + x)^n$ is:
(A) n+1 (B) n -1 (C) n (D) 2n

29. $\binom{3}{3}+\binom{4}{3}+\binom{5}{3}+\text{-------}+\binom{n+2}{3}$ equals:
(A) $\binom{n+3}{3}$ (B) $\binom{n+4}{3}$ (C) $\binom{n+4}{2}$ (D) $\binom{n+3}{4}$

30. The sum of exponent of a and b in every term in the expansion of $(a+b)^n$ is
(A) 1 (B) n (C) n+1 (D) n-1

31. The co-efficient of last term in expansion of $(a-b)^5$ is ________
(A) 1 (B) -1 (C) 0 (D) 2

32. In expansion of $(a + b)^7$, the 2nd term is
(A) a^7 (B) $7a^6b$ (C) $7ab^6$ (D) 0

33. The coefficient of last term in the expansion of $(a+b)^5$ is
(A) 1 (B) -1 (C) 0 (D) 5

34. The middle term in the expansion of $(1 + 2x)^6$ is:
(A) Third (B) Fourth (C) Fifth (D) Sixth

35. Middle term in the expansion of $(x + y)^{21}$ is:
(A) 10th, 11th (B) 9th, 10th (C) 11th, 12th (D) 12th, 13th

36. Sum of exponent of a and b in $(a + b)^n$ in every term equals:
(A) 2n + 1 (B) 2n (C) n (D) n +1

37. If n is even, the Binomial expansions $(a + x)^n$ will have:
(A) One middle term (B) Two middle term (C) Three middle term (D) No middle term

38. The number of terms in the expansion of $(x + b)^8$ is:
(A) 8 (B) 9 (C) 10 (D) 11

Topic III: Binomial Expression:

39. In expansion of $(a + b)^{16}$ middle term will be: (1 time)
(A) 11 (B) 12 (C) 8 (D) 9

40. Expansion of $(1 + x)^{5/2}$ hold when: (4 times)
(A) $|x| < 1$ (B) $|x| > 1$ (C) $|x| = 1$ (D) $|x| \geq 1$

41. Sum of even coefficients in the expansion $(1 + x)^3$ is: (5 times)

(A) 2 (B) 4 (C) 6 (D) 8

42. The terms in the expansion of $(1 - x)^7$ are: (5 times)

(A) 8 (B) 7 (C) 9 (D) 10

43. The expansion of $(1 + 2n)^{-1}$ is valid if: (3 times)

(A) $|x| < 1$ (B) $|x| < \frac{1}{2}$ (C) $|x| = 1$ (D) $|x| < 2$

44. The sum of coefficients in the expansion of $(1 + x)^n$ is:- (2 times)

(A) 2^{n-1} (B) 2^{n-1} (C) 2^{n-2} (D) 2^n

45. $1 - x + x^2 - x^3 + \ldots\ldots\ldots\ldots$ (2 times)

(A) $(1 + x)^{-1}$ (B) $(1 - x)^{-1}$ (C) $(1 + x)^2$ (D) $(1 - x)^2$

46. The second term in the expansion of $(1 - 2x)^{1/2}$ is: (2 times)

(A) x (B) -x (C) x^2 (D) 2x

47. Sum of even coefficient in the binomial expansion $(1 + x)^n$ is: (3 times)

(A) 2^{n-1} (B) 2^n (C) 2^{n+1} (D) $2^n - 1$

48. The sum of binomial co-efficient in the expansion of $(1 + x)^4$ is: (2 times)

(A) 8 (B) 10 (C) 16 (D) 32

49. The expansion of $(1 + 3)^{\frac{1}{2}}$ is valid only when:

(A) $|x|<\frac{1}{2}$ (B) $|x|<\frac{1}{3}$ (C) $|x|< 1$ (D) $|x| = 1$

50. The sum of coefficients in the expansion of $(1+x)^4$ is:

(A) 16 (B) -16 (C) 8 (D) 32

51. Sum of odd co-efficient in the expansion $(1 + x)^n$ is: (2 times)

(A) n^2 (B) 2^{n-2} (C) 2^{n-1} (D) 2^n

52. The sum of odd coefficients in the expansion of $(1 + x)^5$:

(A) 5 (B) 16 (C) 25 (D) 32

53. Expansion of $(1 - 2x)^{\frac{1}{3}}$ is valid if: (3 times)

(A) $|x| < 1$ (B) $|x| < \frac{1}{3}$ (C) $|x| < 2$ (D) $|x| < \frac{1}{2}$

54. Expression of $(3 - 5x)^{1/2}$ is valid if: (2 times)

(A) $|x| < 5$ (B) $|x| < \frac{5}{3}$ (C) $|x| < \frac{3}{5}$ (D) $|x|<\frac{1}{2}$

55. If n is a negative integer than n is:

(A) 1 (B) Not defined (C) Zero (D) n

56. Expansion of $(1+x)^{-1/4}$ is valid only if:

(A) $|x|>1$ (B) $|x|<1$ (C) $|x|<-1$ (D) $|x|>-1$

57. Number of terms in the expansion of $(1 + x)^{2n+1}$ is:

(A) 2n + 1 (B) 2n (C) 2n + 2 (D) 3n + 1

58. If n is not a natural number then the expansion $(1 + x)^n$ is valid if:

(A) $|x| < 2$ (B) $|x| < 1$ (C) $|x| \leq 2$ (D) $|x| > 1$

59. The number of terms in the expansion of $(x^2 - 1)^7$ is:

(A) 2 (B) 7 (C) 8 (D) 12

60. Number of term in the expansion of $(1 + x)^{\frac{1}{3}}$ is: (2 times)

(A) 6 (B) 7 (C) $\frac{1}{3}$ (D) infinite

61. The 2^{nd} term in the expansion of $(1 + 2x)^{\frac{1}{2}}$ is:

(A) x (B) $\frac{2}{15}$ (C) $\frac{1}{2}$ (D) $4x$

2018

62. Expansion of $(1+2x)^{\frac{1}{5}}$ is valid if:

(a) $|x|<1$ (b) $|x|<2$ (c) $|x|<\frac{1}{2}$ (d) $|x|\leq 1$

63. The expression n^2-n+41 respects a prime number for $n\in N$ where
(a) $n\leq 10$ (b) $n\leq 20$ (c) $n\leq 40$ (d) $n\leq 5$

64. In the expansion of $(1+x)^{-3}$ the 4th term is:
(a) $-3x$ (b) $-10x^3$ (c) $6x^2$ (d) $10x^3$

65. If $n\notin z^+$ and $|x|<$ then the expansion $1+nx+\frac{n(n-1)}{2!}x^2+$ __________ is:
(a) Arithmetic (b) Geometric Series (c) Harmonic Series (d) Binomial Series

66. The 2nd term in the expansion $(1+2x)^{-\frac{1}{3}}$ is:
(a) $-\frac{2}{3}x$ (b) $\frac{2}{3}x$ (c) $-6x$ (d) $\frac{x}{3}$

67. The inequality $n!>2^n-1$ is valid if n is:
(a) $n=3$ (b) $n\leq 3$ (c) $n>3$ (d) $n\geq 3$

68. In the expansion of $(x+y)^8$, middle term is:
(a) T_4 (b) T_6 (c) T_3 (d) T_5

69. If n is a positive integer, then $\binom{n}{1}+\binom{n}{3}+\binom{n}{5}+\ldots\ldots+\binom{n}{n-1}$ is equal to.
(a) 2^n (b) 2^{n+1} (c) 2^{n-1} (d) 3^n

70. Show that $ni>n^2$ is true for integral values of: **(2 times)**
(a) $n=3$ (b) $n<4$ (c) $n\geq 4$ (d) $n\leq 4$

2019

71. Francesco Mourollico devised the method of:
(a) Partial fraction (b) Induction (c) Logarithms (d) Binomial

72. If 'n' is positive integer, then n^3+n is divided by:
(a) 2 (b) 3 (c) 4 (d) 5

73. The number of terms in the expansion $(x-3)^{10}$ is:
(a) 10 (b) 11 (c) 12 (d) 13

74. If n is any positive integer then $2^n>2(n+1)$ is true for all:
(a) $n\leq 3$ (b) $n<3$ (c) $n\geq 3$ (d) $n>3$

75. The statement $4^k>3^k+4$ is true for:
(a) $k<2$ (b) $k\leq 2$ (c) $k\neq 2$ (d) $k\geq 2$

76. Total number of terms in expansion of $\left(\frac{x}{2}-\frac{2}{x^2}\right)^{16}$ are:
(a) 17 (b) 16 (c) 15 (d) 14

77. The statement $4^n>3^n+4$ is true if:
(a) $n<2$ (b) $n\neq 2$ (c) $n\geq 2$ (d) $n\leq 2$

78. In the expansion of $(3-2x)^8$, 5th term will be its:
(a) Last term (b) 2nd last term (c) 3rd last term (d) Middle term

79. Third term in the expansion of $(1-2x)^{1/3}$ is equal to:
(a) $-9x^2/4$ (b) $9x^2/4$ (c) $4x^2/9$ (d) $-4x^2/9$

80. Sum of even co-efficient in the expansion of $(1+x)^n$ equals:
(a) 2^{n+1} (b) 2^{n-1} (c) 2^n (d) 2^{1-n}

81. The sum of coefficients in the expension of $(1+x)^5$ is:
(a) 8 (b) 16 (c) 32 (d) 64

82. The sum of odd coefficients in the expansion $(1+x)^n$ is:
(a) n^2 (b) 2^{n-1} (c) 2^n (d) 2^{n-2}

83. Expansion of $(3-5x)^{\frac{1}{2}}$ is valid if:

(a) $|x|<\frac{3}{5}$ (b) $|x|<\frac{5}{3}$ (c) $|x|<5$ (d) $|x|<3$

84. Sum of exponents of a and b in every term of $(a+b)^6$ is:

(a) 6 (b) 7 (c) 3 (d) 12

2021

85. In the middle term T_{r+1} of the binomial expansion of $(a+b)^{12}$, r =

(a) 6 (b) 7 (c) 5 (d) 12

86. The number of terms in the binomial expansion $(a+x)^6$ are

(a) 7 (b) 6 (c) 5 (d) 4

87. Middle Term (S) of $(a+b)^{11}$ is/are

(a) 6th (b) 5th & 6th (c) 6th & 7th (d) 5th

88. 2nd term in the expansion of $(1-x)^{-1}$ is:

(a) 1 (b) 2x (c) x (d) -x

89. The 2nd term in the expansion of $(1-2x)^{\frac{1}{3}}$ is:

(a) $-\frac{2}{3}x$ (b) $\frac{2}{3}x$ (c) $\frac{4}{9}x^2$ (d) $\frac{3}{2}x$

ANSWERS TO THE MULTIPLE CHOICE QUESTIONS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
B	C	C	B	B	B	C	D	B	A	A	C	A	A	B
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
B	B	A	C	A	C	D	A	B	B	D	B	C	D	B
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
B	B	A	B	C	C	A	B	D	A	B	A	B	D	A
46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
B	A	C	C	A	C	B	D	B	D	B	C	B	C	D
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
A	C	D	B	D	A	C	D	C	C	B	A	B	D	D
76	77	78	79	80	81	82	83	84	85	86	87	88	89	
A	C	D	D	B	C	B	A	A	B	A	C	C	A	

SHORT QUESTION'S OF CHAPTER-8 IN ALL PUNJAB BOARDS 2011-2021

Topic I: Mathematical Induction:

1. Show that the inequality $4^n > 3^n + 4$ is true for n = 2 and 3. (4 times)

Sol. $4^n > 3^n + 4 \quad \longrightarrow \quad (1)$

Putting n = 2 and 3 in eq. (1)

Let n = 2

$4^2 > 3^2 + 4$

$16 > 9 + 4$

$16 > 13$

Statement in true for n = 2

Let n = 3

$4^3 > 3^3 + 4$
$64 > 27 + 4$
$64 > 31$

Hence Statement is also true for n = 3

2. Check divisibility of $5^n - 2^n$ by 3 for n = 1,2. **(2 times)**

Sol. Let $a_n = 5^n - 2^n \longrightarrow (1)$

Put n = 1 in (1)

$a_1 = 5 - 2 \quad = 5 - 2 = 3$

Which is divisible by 3 so it is true form = 1

Now put n = 2 in (1)

$a^2 \quad = 5^2 - 2^2 = 25 - 4 = 21$

Which is divisible by 3 so it is true form = 1

3. Using Mathematical Induction, prove that 2+4+6+--------+2n = n (n+1) for n = 1 and n = 2.

Sol. $2 + 4 + 6 + \text{-------} + 2n = n(n+1)$

For n = 1

L.H.S = 2

R.H.S = n(n+1) = 1 (1+1) = 2

L.H.S = R.H.S , So true for n = 1

Now , For n =2

L.H.S = 2+4 = 6

R.H.S = n (n + 1) Put n = 2

= 2(2 + 1)= 2(3) = 6

L.H.S = R.H.S, so it is true for n = 2

4. Show that 1+5+9+---------+ (4n – 3) = n (2n – 1) for integral values of n>0.

Sol. $1 + 5 + 9 + \text{-------} + (4n - 3) = n(2n - 1)$

Case – I: Put n = 1 , then

L.H.S = 4(1) -3 = 4 – 3 = 1

R.H.S = 1 [2 (1) – 3 = 4 – 3 = 1

L.H.S = R.H.S

Case – II Put n = k then

$1 + 5 + 9 + \text{-------}(4k - 3) = k(2k - 1)$

True for n = k

Case – III For n = k + 1

Adding (4k + 1) on both sides

$1 + 5 + 9 + \text{--------} + (4k - 3) + (4k + 1) = k(2k - 1) + (4k + 1)$
$= 2k^2 - k + 4k + 1$
$= 2k^2 + 3k + 1$
$= 2k^2 + 2k + k + 1$
$= 2k(k + 1) + 1(k + 1)$
$= (k + 1)(2k + 1)$
$= (k + 1)[(k + 1) - 1]$

5. Prove the formula $1 + 2 + 4 + \text{--------} + 2^{n-1} = 2^n - 1$ for n = 1 , 2. (4 times)

Sol. $1 + 2 + 4 + \text{-------} + 2^{n-1}$ for n = 2, 2.

Let S_n be the given statement then

S_n: $1 + 2 + 4 + \text{--------} + 2^{n-1} = 2^n - 1 \quad \rightarrow (1)$

For n = 1

L.H.S = 1 (1^{st} term)

R.H.S $= 2^1 - 1 = 1$

L.H.S = R.H.S

For n = 2

L.H.S = 1 + 2 (Sum of 1^{st} 2 terms)

= 3

R.H.S $= 2^2 - 1 = 4 - 1 = 3$
L.H.S = R.H.S (Proved)

6. **Verify the statement $1 + 3 + 5 + ... + (2n - 1) = n^2$ for $n = 3$** **(2 times)**

Sol. $1 + 3 + 5 + ... + (2n - 1) = n^2 \rightarrow (1)$
L.H.S = 1 + 3 + 5 = 9 (sum of 1st 3 terms)
R.H.S $= 3^2 = 9$
L.H.S = R.H.S

7. **Show that $n^3 - n$ is divisible by 6 for n = 2,3.**

Sol. $n^3 - 3 : n = 2,3$
For n = 2
$n^3 - n = (2)^3 - 2 = 8 - 2 = 6$
which is divisible by 6
for n = 3
$n^3 - n = (3)^3 - 3 = 27 - 3 = 24$
which is also divisible by 6

8. **Prove : $1 + 3 + 5 + ... + (2n - 1) = n^2$ for n = 12** **(2 times)**

Sol. $1 + 3 + 5 + ... + (2n - 1) = n^2$
For n = 12
L.H.S = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23
= 144
R.H.S $= n^2 = (12)^2 = 144$
L.H.S = RH.S.

9. **$2 + 6 + 8 + ------- + 2 \times 3^{n-1} = 3^n - 1$ is true for n = 1 , n = 2**

Sol. True For n = 1
$2 + 6 + 8 + ----- + 2 \times 3^{n-1} = 3^n - 1$
$n = 1 \Longrightarrow$ $2 = 3^1 - 1$
$2 = 3 - 1$
2 = 2 True
True For n = 2
$2 + 6 + 8 + ------ + 2 \times 3^{n-1} = 3^n - 1$
$n = 2 \Longrightarrow 2 + 6 = 3^2 - 1$
8 = 9 – 1
8 = 8 Proved True for n = 2
Hence Proved
Given formula is True for n = 2

10. **Prove that $n^2 > n + 3$ for integral value of n = 3 , 4**

Sol. Given Statement $n^2 > n + 3$
$n = 3 \Longrightarrow$ $3^2 > 3 + 3$
9 > 6 True
$n = 4 \Longrightarrow$ $4^2 > 4 + 3$
16 > 7 True
Hence Proved
Given Statement is True for $n = 3_1 4$

11. **Prove the formula $r + r^2 + r^3 + ---- + rn = \frac{r(1-r^n)}{1-r}$ for n = 1 and n = 2**

Sol. For n = 1
$r + r^2 + r^3 + ------ + r^n = \frac{r(1-r^n)}{1-r}$
$n = 1 \Longrightarrow r = \frac{r(1-r^1)}{1-r}$
r = r (1)
r = r True
for n = 2
$r + r^2 + r^3 + -------+ r^n = \frac{r(1-r^n)}{1-r}$
$n = 2 \Longrightarrow r + r^2 = \frac{r(1-r^n)}{1-r}$
$r(1 + r) = \frac{r(1+r)(1-r)}{(1-r)}$

$\Rightarrow r(1+r) = r(1+r)$
$\Rightarrow$ True for n = 2

12. Show that $n! > n^2$ is True for n = 4, 5. **(2 times)**

Sol. $n! > n^2$
$n = 4 \Rightarrow 4! > 4^2$
24 > 16 True
$n = 5 \Rightarrow 5! > 5^2$
120 > 25 True
Hence Proved
Given Statement is True for n = 4 and n = 5

13. Prove the formula. **(3 times)**

$1 + 3 + 5 + 7 + \text{----} + (2n-1) = n^2$ for n = 1 , 2
Proof
For n = 1
$1 + 3 + 5 + 7 + \text{-------} + (2n-1) = n^2$
$n = 1 \Rightarrow 1 = 1^2$
1 = 1 True
For n = 2
$1 + 3 + 5 + 7 + \text{-------} + (2n-1) = n^2$
$n = 2 \Rightarrow 1 + 3 = 2^2$
4 = 4 True
Hence Proved Given formula is True for n = 1 , 2

14. Check, $1 + 2 + 4 + \text{-----} + 2^{n-1} = 2^n - 1$ for n = 1 , 2. **(2 times)**

Sol Check For n = 1
$1 + 2 + 4 + \text{------} + 2^{n-1} = 2^n - 1$
$n = 1 \Rightarrow 1$
$1 = 2^1 - 1$
$1 = 2 - 1$
1 = 1 True
For n = 2
$1 + 2 + 4 + \text{-----} + 2^{n-1} = 2^n - 1$
$n = 2 \Rightarrow 1 + 2 = 2^2 - 1$
3 = 4 – 1
3 = 3 True
Proved , Given formula is True for n = 1 , 2

15. Prove that $n! > 2^n - 1$ for n = 4 , 5. **(2 times)**

Sol. For n = 4
$n! > 2^n - 1$
$n = 4 \Rightarrow 4! > 2^4 - 1$
$4 \times 3 \times 2 \times 1 > 16 - 1$
24 > 15 True
For n = 5
$n! > 2^n - 1$
$n = 5 \Rightarrow 5! > 2^5 - 1$
$5 \times 4 \times 3 \times 2 \times > 32 - 1$
120 > 31 True
Proved , $n! > 2^n - 1$ is
True for n = 4 , 5

16. Show that $4^n > 3^n + 4$ is not true for n = 1.

Sol **Given**
$4^n > 3^n + 4$ at n=1
$(4)^1 > (3)^1 + 4$
4 > 3 + 4
4 > 7
Which is not for n = 1

Topic II: Binomial Theorem with Positive Integers:

17. Evaluate $(9.9)^3$. **(2 times)**

Sol. $(9.9)^3 = (10 - 0.1)^3$

$= \binom{3}{0}(10)^3 + \binom{3}{1}(10)^2(-0.1) + \binom{3}{2}(10)(-0.1)^2 + \binom{3}{3}(10)^0(-0.1)^3$
$= 1000 - 30 + 0.3 - 0.001$
$= 970.299$

18. Expand $(x^2 + x - 1)^2$ in descending powers of x.

Sol. $[x^2 + (x-1)]^2 = \binom{2}{0}(x^2)^2 + \binom{2}{1}(x^2)^1(x-1) + \binom{2}{2}(x^2)(x-1)^2$
$= (1)x^4 + 2x^2(x-1) + 1(1)(x^2 - 2x + 1)$
$= x^4 + 2x^3 - 2x^2 + x^2 - 2x + 1$
$= x^4 + 2x^3 - x^2 - 2x + 1$

19. State any four points of the observation in the expansion of $(a + x)^n$.

Sol. 1- The number of terms in the expansion is one greater than its index.
2- The sum of exponents of a and x in each term of the expansion is equal to its index.
3- The exponent of a decreases from index to zero.
4- The exponent of x increases from zero to index.

20. State the Binomial theorem. **(3 times 2018)**

Sol. The rule of formula for expansion of a binomial raised to any +ve integral power 'n' is called the binomial theorem for positive integral index n. For any +ve integer n.

$(a + x)^n = \binom{n}{o}a^n + \binom{n}{1}a^{n-1}x + \binom{n}{2}a^{n-2}x^2 + \ldots\ldots\ldots + \binom{n}{n-1}ax^{n-1} + \binom{n}{n}x^n$

21. Use Binomial Theorem to expand $(a + 2b)^4$.

Sol. $= \binom{4}{0}a^4 + \binom{4}{1}a^3(2b) + \binom{4}{2}a^2(2b)^2 + \binom{4}{3}a^1 2ab + \binom{4}{4}(2b)^1$

$= a^4 + \frac{4!}{1!(4-1)!}a^3.2b + \frac{4!}{2!2!}a.4b^2 + \frac{4!}{3!1!}a.8b^2$

$= a^4 + \frac{4.3.2.1}{3.2.1}2a^2b + \frac{4.3.2!}{2.1.2!}4a^2b^2 + \frac{4.3!}{3!}8ab^3 + 16b^4$

$= a^4 + 8a^3b + 6(4a^2b^2) + 32ab^3 + 16b^4$

$= a^4 + 8a^3b + 24a^2b^2 + 32ab^3 + 16b^4$

22. What is Binomial series? Write its formula for expansion.

Sol. When n is a negative integer or fraction then

$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots\ldots\ldots\ldots$

Provided $|x| < 1$ The series of the type

$1 + nx + \frac{n(n-1)}{21}x^2 + \frac{n(n-1)(n-2)}{31}x^3 + \ldots\ldots\ldots\ldots\ldots$

Is called binomial series.

23. Find the terms involving x^{-2} in the expansion of $\left(x - \frac{2}{x^2}\right)^{13}$.

Sol. $\left(x - \frac{2}{x^2}\right)^{13}$

$T_{r+1} = \binom{n}{r}a^{n-r}b^r$

Here $a = x$, $b = -\frac{2}{x^2}$, $n = 13$, then

$= \binom{13}{r}(x)^{13-r}\left(-\frac{2}{x^2}\right)^r = \binom{13}{r}x^{13-r}(-2)^r(x^{-2})^r$

$= \binom{13}{r}x^{13-r-2r}(-2)^r = \binom{13}{r}x^{13-3r}.(-2)^r \quad \rightarrow(1)$

For x^{-2}, Put $13 - 3r = -2$
$13 + 2 = 3r$
$15 = 3r$
$\frac{15}{3} = r$

$5 = r$

Then (1) becomes

$T_{5+1} = \binom{13}{r} x^{13-3(5)} . (-2)^5$

$= \frac{13!}{5!(13-5)} x^{13-15} (-32)$

$= \frac{13.12.11.10^2.9.8!}{5.4.3.2.1.8!} x^{-2} (-32) = 13.11.(-32)\, x^{-2} = -41184\, x^{-2}$

24. Using Binomial expansion, expand $(2a - {}^{x}/_{a})^3$.

Sol. $(2a - {}^{x}/_{a})^3$

$= (2a)^7 + \binom{7}{1}(2a)^6\left(-\frac{x}{a}\right) + \binom{7}{2}(2a)^5\left(-\frac{x}{a}\right)^2 + \binom{7}{3}(2a)^4\left(-\frac{x}{a}\right)^3 + \binom{7}{4}(2a)^3$
$\left(-\frac{x}{a}\right)^4 + \binom{7}{5}(2a)^2\left(-\frac{x}{a}\right)^5 + \binom{7}{7}(2a)^0.\left(-\frac{x}{a}\right)^7$

$= 128\,a^7 + 7\,.\,64a^6\left(-\frac{x}{a}\right) + \frac{7.6}{2.1}(32a^5)\frac{x^2}{a^2} + \frac{7.6.5}{3.2.1}(16a^4)\left(-\frac{x^3}{a^3}\right) + \frac{7.6.5}{3.2.1}(8a^3)\frac{x^4}{a^4} +$
$\frac{7.6}{2.1}(4a^2)\left(-\frac{x^5}{a^5}\right) + 7(2a)\frac{x^6}{a^6} + 1.1\left(-\frac{x^3}{a^7}\right)$

$= 128a^7 + 448a^5 x + 672a^3x^2 - 560\,ax^3 + 280\frac{x^4}{a} - 84\frac{x^5}{a^3} + 14.\frac{x^6}{a^5} - \frac{x^7}{a^7}$

25. Write the general term of binomial expansion. (2 times)

Sol. General Terms of Binomial Expansion.

The (r + 1)th term in the expansion of $(a + x)^n$ is ${}^nc_r.\, a^{n-r}.x^r$ and we denote it as T_{r+1} i.e.

$T_{r+1} = \binom{n}{r} a^{n-r}x^r$ called the general term of the binomial expansion of $(a + x)^n$

26. Find 5^{th} term in the expansion of $\left(\frac{3}{2}x - \frac{1}{3x}\right)^{11}$. (2 times)

Sol. $\left(\frac{3}{2}x - \frac{1}{3x}\right)^{11}$

Here $a = \frac{3x}{2}$, $b = -\frac{1}{3x}$

$n = 11$, $r = 4$

$T_{r+1} = \binom{n}{r} a^{n-r}.b^r.$

$T_{4+1} = \binom{11}{4}\left(\frac{3x}{2}\right)^{11-4}.\left(-\frac{1}{3x}\right)^4$

$T_5 = \frac{11!}{4!7!}\left(\frac{3x}{2}\right)^7.\frac{1}{81x^4}$

$T_5 = \frac{11.10.9.8}{4.3.2.1}.\frac{27x^3}{128} = \frac{7920}{24}.\frac{27x^3}{128} = \frac{330 \times 27.x^3}{128}$

$T_5 = \frac{8910}{128}x^3 = \frac{4455}{64}x^3$

27. $\binom{3}{3} + \binom{4}{3} + \binom{5}{3} + \cdots + \binom{n+2}{3} = \binom{n+3}{4}$ is true for n = 1,2

Sol. $\binom{3}{3} + \binom{4}{3} + \binom{5}{3} + \cdots + \binom{n+2}{3} = \binom{n+3}{4}$

$n = 1 \Rightarrow \binom{3}{3} = \binom{1+3}{4}$

$\binom{3}{3} = \binom{4}{4}$

1 = 1 True

$n = 2 \Rightarrow \binom{3}{3} + \binom{4}{3} = \binom{2+3}{4}$

$\binom{3}{3} + \binom{4}{3} = \binom{5}{4}$

$1 + 4 = 5$
$5 = 5$ True
Proved

28. **Given statement is True for n = 1 , 2 Expand $\left(3a - \frac{x}{3a}\right)^4$ (2 times 2018)**

Sol. $\left(3a - \frac{x}{3a}\right)^4$

$= \binom{4}{0}(3a)^4\left(\frac{-x}{3a}\right)^0 + \binom{4}{1}(3a)^3\left(\frac{-x}{3a}\right)^1 + \binom{4}{2}(3a)^2\left(\frac{-x}{3a}\right)^2 + \binom{4}{3}(3a)^1\left(\frac{-x}{3a}\right)^3 +$
$\binom{4}{4}(3a)^0\left(\frac{-x}{3a}\right)^4$

$= (1)\,81a^4(1) + 4\,(27a^3)\left(\frac{-x}{3a}\right) + 6\,(9a^2)\frac{x^2}{9a^2} + 4(3x)\left(\frac{-x^3}{27a^3}\right) + 1\,(1)\left(\frac{x^4}{81a^4}\right)$

$= 81a^4 - 36a^2x + 6x^2 - \frac{4x^3}{9a^2} + \frac{x^4}{81a^4}$

29. **Find the 6th Term of $\left(x^2 - \frac{3}{2x}\right)^{10}$ (5 times)**

Sol. $a = x^2$, $b = \frac{-3}{2x}$, $n = 10$

$T_r + 1 = \binom{n}{r} a^{n-r} b^r$

$T_{5+1} = \binom{10}{5}(x^2)^{10-5}\left(\frac{-3}{2x}\right)^5$

$T_6 = 256\,(x^2)^5\left(-\frac{243}{32x^5}\right)$

$T_6 = 256x^{10}.\left(-\frac{243}{32x^5}\right) = -\frac{62208\,x^5}{32} = -1944x^5$

30. **Write T_{r+1} Term of Binomial Theorem in the expansion of $(a + x)^n$**

Sol. $(a + x)^n$

$a = a$, $b = x$

$n = n$

We know $T_{r+1} = \binom{n}{r} a^{n-r} . b^r$

$T_{r+1} = \binom{n}{r} (a)^{n-r} (x)^r$

Which is required Term

31. **Calculate $(9.98)^4$ By using Binomial Theorem. (2 times 2018)**

Sol. $(9.98)^4 = (10 - 0.02)^4$

Now $(10 - 0.02)^4$

$= \binom{4}{0}(10)^4(-0.02)^0 + \binom{4}{1}(10)^3(-0.02)^1 + \binom{4}{2}(10)^2(-.02)^2 + \binom{4}{3}(10)^1(-.02)^3 + \binom{4}{4}$
$(10)^0(-0.02)^4$

$= 1(10000)\,(1) + 4\,(100)\,(-0.02) + 6\,(100)\,(0.0004) + 4\,(10)\,(-0.000008) + 1\,(1)$
(0.00000016)

$= 10000 - 80 + 0.24 - 0.00032 + 0.00000016 = 9920.24$

Hence proved Given statement is True for n = 1 and n = 2

32. **Find the sixth Term in the expansion of $\left(x - \frac{2}{x}\right)^{10}$**

Sol. $a = x$, $b = \frac{-2}{x}$, $n = 10$ $T6 = ?$

We know

$T_{r+1} = \binom{n}{r} a^{n-r}.b^r$

$T_{5+1} = \binom{10}{5} (x)^{10-5} \left(\frac{-2}{x}\right)^5$

$T_6 = (252)(-32)$

$T_6 = -8064$

33. Simplify $(2+i)^5 =$

Given $(2+i)^5 =$

Using binomial theorem

$$(2+i)^5 = \binom{n}{0} a^n x^0 + \binom{n}{1} a^{n-1} x^1 + \binom{n}{2} a^{n-2} x^2 + \dots\dots + \binom{n}{n} a^0 x^n$$

$$(a+x)^5 = \binom{5}{0}(2)^5(i)^0 + \binom{5}{1}(2)^4(i)^1 + \binom{5}{2}(2)^3(i)^2 + \binom{5}{3}(2)^2(i)^3 + \binom{5}{4}(2)^1(i)^4 + \binom{5}{5}(2)^0 i^5$$

$$= 1(32)(1) + 5(16)i + 10(8)i^2 + 10(4)i^3 + 5(2)i^4 + 1.i^5$$

$$= 32 + 8i + 80(-1) + 40i + 10(1) + i \qquad \because i^2 = -1$$

$$= 32 + 8i - 80 + 40i + 10 + i$$

$$= 49i - 38$$

34. Calculate $(2.02)^4$ by means of binomial theorem. **(4 times 2018)**

Sol $(2.02)^4$

Using binomial theorem

$(2.02)^4 = (2 + 0.02)^4$

$$= \binom{4}{0}(2)^4(0.02)^0 + \binom{4}{1}(2)^3(0.02)^1 + \binom{4}{2}(2)^2(0.02)^2 + \binom{4}{3}(2)^1(0.02)^3 + \binom{4}{4}(2)^0(0.02)^4 =$$

$(1)(2)^4(1) + 4(2)^3(0.02) + 6(4)(0.0004) + 4(2)(0.000008) + (1)(1)(0.00000016)$

$= 16 + 0.64 + 0.0096 + 0.000064 + 0.00000016$

$= 16.64966416$

Topic III: Binomial Expression:

35. Find the co-efficient of x^3 in the expansion of $\frac{1+x^2}{(1+x)^2}$

Sol. $\frac{1+x^2}{(1+x)^2} = (1+x^2)(1+x)^{-2}$

$$=(1+x^2)\left[1 + (-2)(x) + \frac{-2(-2-1)}{2!}x^2 + \frac{-2(-2-1)(-2-2)}{3!}x^3 + \dots\right]$$

$= (1+x^2)[1 - 2x + 3x^2 - 4x^3 + \dots]$

Terms involving x^3

$= -4x^3 - 2x^3 = -6x^3$

Co-efficient of $x^3 = -6$

36. If x is so small that its square and higher powers be neglected, then show that: $\frac{\sqrt{4-x} + (8-x)^{1/3}}{(8-x)^{1/3}} = 2 - \frac{1}{12}x$.

Sol. $\frac{\sqrt{4-x} + (8-x)^{1/3}}{(8-x)^{1/3}}$

$$= \left[(4-x)^{1/2} + (8-x)^{1/3}\right](8-x)^{-1/3}$$

$$= \left[4^{1/2}(1 - x/4)^{1/2} + 8^{1/3}(1 - x/8)^{1/3}\right](8)^{-1/3}\left(1 - \frac{x}{8}\right)^{-1/3}$$

neglecting square and higher powers.

$= \left[2\left(1+\frac{1}{2}(-x/4)\right) + 2\left(1+\frac{1}{3}(-x/8)\right)\right].2^{-1}\left(1+\left(-\frac{1}{3}\right)\left(-\frac{x}{8}\right)\right)$

$= [2(1-{}^{x}/_{8}) + 2(1-{}^{x}/_{24})]\frac{1}{2}\left(1+\frac{x}{24}\right)$

$= 2.\frac{1}{2}[2(1-{}^{x}/_{8}) + (1-{}^{x}/_{24})]\left(1+\frac{x}{24}\right)$

$= \left[1-\frac{x}{8}+1-\frac{x}{24}\right][1+{}^{x}/_{24}]$

$= \left[2-\frac{4x}{24}\right][1+{}^{x}/_{24}]$

$= \left[2-\frac{x}{6}\right]\left[1+\frac{x}{24}\right] = 2+\frac{x}{12}-\frac{x}{6} = 2-\frac{x}{12}$ = R.H.S

37. Expand $(2-3x)^{-2}$ upto four terms. **(4 times)**

Sol. $(2-3x)^{-2} = (2)^{-2}\left(1-\frac{3}{2}x\right)^{-2} = \frac{1}{4}\left(1-\frac{3}{2}x\right)^{-2}$

$$= \frac{1}{4}\left[1+(-2)\left(-\frac{3}{2}x\right)+\frac{(-2)(-2-1)}{2!}\left(\frac{-3}{2}x\right)^2 + \frac{-2(-2-1)(-2-2)}{3!}\left(\frac{-3}{2}x\right)^3 + \frac{-2(-2-1)(-2-2)}{3!}\left(\frac{-3}{2}x\right)^4+\cdots\right]$$

$$= \frac{1}{4}\left[1+3x+\frac{27}{4}x^2+\frac{27}{2}x^3+\frac{405}{16}x^4+\ldots\right]$$

$$= \frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^2+\frac{27}{8}x^3+\frac{405}{64}x^4$$

38. Evaluate $(0.98)^{1/2}$ by mean of Binomial Theorem to three places or decimal. **(2 times)**

Sol. $(0.98)^{1/2} = (1-0.02)^{1/2}$

$(0.98)^{1/2} = 1+\frac{1}{2}(-0.02)+\frac{{}^{1}/_{2}({}^{1}/_{2}-1)}{2!}(-0.02)^2+\frac{{}^{1}/_{2}({}^{1}/_{2}-1)({}^{1}/_{2}-2)}{3!}(-0.02)^3+\ldots$

$= 1-0.01-\frac{1}{8}(0.00004)\ldots$

$= 1-0.01-0.001-\ldots$

$= 0.989$

39. Expand $(1+x)^{-1/3}$ upto 4 terms. **(4 times)**

Sol. $(1+x)^{-1/3}$

$\because (1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+---$

$(1+x)^{-1/3} = 1+\left(-\frac{1}{3}\right)x\,\frac{\left(-\frac{1}{3}\right)\left(-\frac{1}{3}-1\right)}{2!}x^2+\frac{\left(-\frac{1}{3}\right)\left(-\frac{1}{3}-1\right)\left(-\frac{1}{3}-2\right)}{3!}x^3---$

$= 1-\frac{x}{3}+\frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2}x^2+\frac{\left(-\frac{1}{3}\right)\left(-\frac{1}{4}\right)\left(-\frac{7}{3}\right)}{3.2}x^3---$

$= 1-\frac{x}{3}+\frac{2}{9}x^2-\frac{14}{81}x^3+---$

Hence it is true for all +ve integer n.

40. Expand upto 4 terms $(8-2x)^{-1}$ **(3 times)**

Sol. $(8-2x)^{-1}$

$= (8)^{-1}\left(1-\frac{2}{8}x\right)^{-1} = \frac{1}{8}\left(1-\frac{1}{4}x\right)^{-1}$

$= \frac{1}{8}\left\{1+(-1)\left(-\frac{1}{4}x\right)+\frac{(-1)(-1-1)}{2!}\left(-\frac{1}{4}x\right)^2+\frac{(-1)(-1-1)(-1-2)}{3!}\left(-\frac{1}{4}x\right)^3+\ldots\right\}$

$= \frac{1}{8}\left\{1+\frac{x}{4}+\frac{x^2}{16}+\frac{x^3}{64}+\cdots\right\}$

41. Expand $(1+2x)^{-1}$ upto 4 terms. **(4 times 2018)**

Sol. $(1+2x)^{-1}$

$(1+2x)^{-1} = 1 + (-1)(2x) + \frac{(-1)(-1-1)}{2!}(2x)^2 + \frac{(-1)(-1-1)(-1-2)}{3!}(2x)^3 \cdots$

$= 1 - 2x + 4x^2 - 8x^3 + \cdots$

42. If x is so small that its cube and higher powers can be neglected, then show that $\sqrt{1-x-2x^2} = 1 - \frac{x}{2} - \frac{9}{8}x^2$.

Sol. $\sqrt{1-x-2x^2} = 1 - \frac{x}{2} - \frac{9}{8}x^2$

L.H.S. $= \sqrt{1-x-2x^2} = (1-x-2x^2)^{1/2}$

$= \{1-(x+2x^2)\}^{1/2}$

$= 1 + \left(\frac{1}{2}\right)(-(x+2x^2)) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}[-(x+2x^2)]^2$

($\because$ By given condition)

$= 1 - \frac{x}{2} - x^2 + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2}(x+2x^2)^2$

$= 1 - \frac{x}{2} - x^2 - \frac{1}{x}x^2$ $\quad\therefore$ neglecting x^3 and higher powers.

$= 1 - \frac{x}{2} - \frac{9}{x}x^2$ = R.H.S

Hence $\sqrt{1-x-2x^2} = 1 - \frac{x}{2} - \frac{9}{8}x^2$

43. Expand $(4-3x)^{\frac{1}{2}}$ upto two terms. **(2 times 2018)**

Sol. $(4-3x)^{\frac{1}{2}} = \left[4\left(1-\frac{3x}{4}\right)\right]^{\frac{1}{2}} = (2^2)^{\frac{1}{2}}\left(1-\frac{3x}{4}\right)^{\frac{1}{2}}$ neglected nest terms

$(4-3x)^{\frac{1}{2}} = 2\left(1-\frac{3x}{4}\right)^{\frac{1}{2}}$ neglected nest terms

$(4-3x)^{\frac{1}{2}} = 2\left[1 + \frac{\frac{1}{2}}{1!}\left(\frac{-3x}{4}\right)\right]$

$(4-3x)^{\frac{1}{2}} = 2\left(1 - \frac{1}{2}\left(\frac{3x}{4}\right)\right)$

$(4-3x)^{\frac{1}{2}} = 2\left(1 - \frac{3x}{8}\right)$

44. Expand $(1-2x)^{\frac{1}{3}}$ upto 4 Terms **(2 times)**

Sol. $(1-2x)^{\frac{1}{3}}$

$= 1 + \frac{\frac{1}{3}}{1!}(-2x) + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}(-2x)^2 + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3!}(-2x)^3 + \cdots$

$(1-2x)^{\frac{1}{3}} = 1 - \frac{1}{3}(2x) + \frac{1}{2\times1}\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)(4x^2) + \frac{1}{3!}\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)\left(\frac{-5}{8}\right)(-8x^3)$

$(1-2x)^{\frac{1}{3}} = 1 - \frac{2}{3}x - \frac{4x^2}{9} - \frac{40x^3}{3\times2\times1\times27} + \cdots$

$(1-2x)^{\frac{1}{3}} = 1 - \frac{2}{3}x - \frac{4}{9}x^2 - \frac{20}{81}x^3 + \cdots$

45. If x be so small that its square and higher power be neglected show that **(3 times)**

$\frac{\sqrt{4+x}}{(1+x)^3} = 2 + \frac{25}{4}x$

Sol. L.H.S $= \frac{(4+x)^{1/2}}{(1-x)^3}$

$= \frac{\left[\frac{4}{4}(4+x)\right]^{\frac{1}{2}}}{(1-x)^3}$

$= \left[4\left(\frac{4}{4}+\frac{x}{4}\right)\right]^{\frac{1}{2}}(1-x)^{-3}$

$L.H.S = (4)^{\frac{1}{2}}\left(1+\frac{x}{4}\right)^{\frac{1}{2}}(1-x)^{-3}$

$= (2^2)^{\frac{1}{2}}\left(1+\frac{x}{4}\right)^{\frac{1}{2}}(1-x)^{-3}$

$= 2\left[1+\frac{\frac{1}{2}}{1!}\left(\frac{x}{4}\right)\right]+\left[1+\frac{-3}{1!}(-x)\right]$

$= 2\left(1+\frac{x}{8}\right)(1+3x) = 2\left(1+3x+\frac{x}{8}+\frac{3x^2}{8}\right)$

$= 2\left(1+\frac{24n+x}{8}+\text{Neglect}\right) = 2\left(1+\frac{25}{8}x\right)$

$= 2+\frac{25}{4}x = R.HS$

46. If x is so small that its square and higher power be neglected show that (5 times)

$\frac{1-x}{\sqrt{1+x}} = 1-\frac{3}{2}x$

Sol. $L.H.S = \frac{1-x}{\sqrt{1+x}}$

$= \frac{1-x}{(1+x)^{1/2}}$

$= (1-x)(1+x)^{\frac{-1}{2}}$

$= (1-x)\left[1+\frac{\frac{-1}{2}}{1!}(x)\right]$

$= (1-x)\left(1-\frac{1}{2}x\right)$

$= 1-\frac{1}{2}x - x+\frac{1}{2}x^2$

$L.H.S = 1-\frac{x}{2}- x+\frac{1}{2}x^2$

$= 1-\left(\frac{x}{2}+x\right)$

$= 1-\left(\frac{x+2x}{2}\right) = 1-\frac{3x}{2} = R.H.S$

47. Calculate $(0.97)^3$ By using Binomial Theorem (8 times)

Sol. $(0.97)^3 = (1-0.03)^3$

$(0.97)^3$

Now $(0.97)^3 = (1-0.03)^3$

$= \binom{3}{0}(1)^3(-0.03)^0 + \binom{3}{1}(1)^2(-0.03)^1 + \binom{3}{2}(1)^1(-0.03)^2 + \binom{3}{3}(1)^0(-0.03)^2$

$= 1(1)(1) + (3)(1)(-0.03) + 3(0.0009) + 1(1)(-0.000027)$

$= 1 - 0.09 + 0.0027 - 0.00027 = 0.9127$

48. Expand upto Four Terms $(1+2x)^{-1}$

Sol: $(1+2x)^{-1}$

$= 1+\frac{-1}{1!}(2x)+\frac{-1(-1-1)}{2!}(2x)^2+\frac{-1(-1-1)(-1-2)}{3!}(2x)^3+\ldots$

$= 1-2x+\frac{-1(-2)}{2\times1}(4x^2)+\frac{-1(-2)(-3)}{3\times2\times1}(8x^3)+\ldots$

$= 1-2x+\frac{2}{2}(4x^2)-\frac{6}{6}(8x^3)+ ----$

$= 1-2x+4x^2-8x^3+ ------+$

49. Neglecting x^2 and higher powers of "x", show that $\sqrt{\frac{1+2x}{1-x}} \approx 1+\frac{3}{2}x$ (4 times)

Sol $\sqrt{\frac{1+2x}{1-x}} \approx 1+\frac{3}{2}x$

Neglecting x^2 and higher powers of x

L.H.S $=\sqrt{\frac{1+2x}{1-x}}$

$$=\frac{(1+2x)^{1/2}}{(1-x)^{1/2}}=(1+2x)^{1/2}(1-x)^{-1/2}$$

$$=\left(1+\frac{1}{2}(2x)\right)\left(1+(-x)\left(-\frac{1}{2}\right)\right)$$

$$=(1+x)\left(1+\frac{x}{2}\right)$$

$$=1+\frac{x}{2}+x+\frac{x^2}{2}$$

$$=1+\frac{x}{2}+x+0$$

$$=1+x+\frac{x}{2}$$

$$=1+\left(\frac{2x+x}{2}\right)=1+\frac{3}{2}x \quad = \quad \text{R.H.S}$$

50. Expand upto three terms $(1-x)^{1/2}$ (4 times)

Sol: Given $(1-x)^{1/2}=(1+(-x))^{-2/3}$

Using binomial series.

$$(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\ldots$$

$$(1+(-x))^{1/2}=1+\frac{1}{2}(-x)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(-x)^2+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-3\right)}{3!}(-x)^3+\ldots$$

$$=1-\frac{x}{2}+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2.1}x^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3.2.1}(-x^3)+\ldots$$

$$=1-\frac{x}{2}-\frac{1}{8}x^2+\frac{3}{48}(-x^3)+\ldots$$

$$=1-\frac{x}{2}-\frac{x^2}{8}-\frac{x^3}{16}+\ldots$$

51. Expand $(8-5x)^{-2/3}$ to two terms.

Sol Given $(8-5x)^{-2/3}$ expand two terms

$$=\left[8\left(1-\frac{5x}{8}\right)\right]^{-2/3}$$

$$= (8)^{-2/3}\left(1-\frac{5x}{8}\right)^{-2/3}$$

$$= (2^3)^{-2/3}\left(1-\frac{5x}{8}\right)^{-2/3}$$

$$= (2)^{-2}\left(1-\frac{5x}{8}\right)^{-2/3}$$

$$= \frac{1}{4}\left(1-\frac{5x}{8}\right)^{-2/3} \qquad \because (1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\ldots$$

$$= \frac{1}{4}\left[1+\left(\frac{-2}{3}\right)\left(\frac{-5x}{8}\right)+\ldots\ldots\right]$$

$$= \frac{1}{4}\left[1+\frac{10x}{24}+\ldots\ldots\right]$$

$$= \frac{1}{4}\left[1+\frac{5x}{12}+\ldots\ldots\right]$$

$$= \frac{1}{4}+\frac{5}{48}x+\ldots\ldots\ldots$$

52. Using binomial theorem find the value of $\sqrt{99}$. (2 times)

Sol Given $\sqrt{99} = (99)^{1/2} = (100-1)^{1/2} = (100)^{1/2}\left(1-\frac{1}{100}\right)^{1/2}$

$$= (10^2)^{1/2}\left[1+\left(\frac{-1}{100}\right)\right]^{1/2}$$

$$= 10\left[1+\frac{1}{2}\left(\frac{-1}{100}\right)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(\frac{-1}{100}\right)^2+\ldots\ldots\right]$$

$$= 10\left[1-\frac{1}{200}+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}\left(\frac{1}{100}\right)^2+\ldots\ldots\right]$$

$$= 10\left[1-\frac{1}{200}+\frac{1}{8!}\left(\frac{1}{1000}\right)+\ldots\ldots\right]$$

$$= 10[1-0.005+0.0000125+\ldots\ldots]$$

$$= 10(0.9949875) = 9.950 \quad \textbf{Ans.}$$

2018

53. Define Mathematical Induction?

Ans: Mathematical induction is a mathematical proof technique. It is essentially used to prove that a property P(n) holds for every natural number n, i.e. for n = 0, 1, 2, ... and so on.

54. Show that $1 + 2 + 4 + \ldots\ldots 2^{n-1} = 2^n - 1$ is true for n = 4, 5.

Sol: Put n = 4

$1+2+4+8=2^4-1$

$15 = 16-1$

$15 = 15$

Since condition is Satisfied for n = 4.

Put n = 5

$1+2+4+8+16=2^5-1$

$31 = 32-1$

$31 = 31$

Since condition is satisfied for n = 5.

55. Show that satisfied n = 1, 2

$$1+\frac{1}{2}+\frac{1}{4}+-----+\frac{1}{2^{n-1}}=2\left(1-\frac{1}{2^n}\right)$$

Sol: Put n = 1

$1=2\left(1-\frac{1}{2^1}\right)$

$=2\left(1-\frac{1}{2}\right)$

$=2\left(\frac{1}{2}\right)$

$=1$

⇨ Condition is satisfied

Now put n = 2

$1+\frac{1}{2}=2\left(1-\frac{1}{2^2}\right)$

$1+\frac{1}{2}=2\left(1-\frac{1}{4}\right)$

$\frac{3}{2} = 2\left(\frac{3}{4}\right)$

$\frac{3}{2} = \frac{3}{2}$

⇨ Condition is satisfied for n = 2.

56. Show that n^2+n is divisible by 2 for n = 2, 3

Sol: Put n = 1 $S(n)=n^2+n$

$S(1)=(1)^2+1$

$=1+1=2$

S (1) is clearly divided by 2 then condition is satisfied.

Now put n = 3

$S(3)=9+3$

$=12$

S(3) is also clearly divided by 2

So condition is true for n = 3.

57. Expand $(2-3x)^{-1}$ upto three terms.

Sol: $(2-3x)^{-1}=2\left(1+\left(\frac{-3}{2}x\right)\right)^{-1}$

By using

$$(1+x)^n = 1 + n(x) + \frac{n(n-1)}{2!}x^2 + ---$$

$$(2-3x)^{-1} = 2\left[1+(-1)\left(\frac{-3}{2}x\right)+\frac{(-1)(-1-1)}{2!}\left(\frac{-3}{2}x\right)^2+----\right]$$

$$(2-3x)^{-1} = 2\left[1+\frac{3}{2}x+\frac{9}{4}x^2+----\right]$$

$$= 2\left[1+3x+\frac{9}{2}x^2+----\right]$$

58. **Find the value of $3\sqrt{65}$ to '2' places of decimal by using Binomial series.**

Sol:

$$3\sqrt{65} = (65)^{1/3}$$

$$= (64-1)^{1/3}$$

$$= (64)^{1/3}\left(1-\frac{1}{64}\right)^{1/3}$$

$$= (64)^{1/3}\left(1-\frac{1}{64}\right)^{1/3}$$

$$= (64)^{1/3}\left[1+\frac{1}{3}\left(-\frac{1}{64}\right)+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}\left(-\frac{1}{64}\right)^2+----\right]$$

$$= 4\left[1-\frac{1}{192}+\frac{-2/9}{2}\left(\frac{1}{4096}\right)+----\right]$$

$$= 4\left[1-\frac{1}{192}-\frac{1}{9}\left(\frac{1}{4096}\right)+---------\right]$$

$$= \left[4-\frac{1}{48}-\frac{1}{9216}+--------\right]$$ Ans.

59. **Using Bionomial Series and find the value of $\frac{1}{5\sqrt{252}}$ to three place of decimal.**

Sol:

$$\frac{1}{5\sqrt{252}}$$

$$= (252)^{-1/5}$$

$$= (243+9)^{-1/5}$$

$$= (243)^{-1/5}$$

$$= \left(1+\frac{9}{243}\right)^{-1/5}$$

$$\frac{1}{5\sqrt{252}} = (3^5)^{-1/5}\left(1+\frac{1}{27}\right)^{-1/5}$$

$$=(3)^{-1}\left[1+\left(-\frac{1}{5}\right)\left(\frac{1}{27}\right)+\frac{\left(-\frac{1}{5}\right)\left(-\frac{1}{5}-1\right)}{2!}\times\left(\frac{1}{27}\right)^2+------\right]$$

$$=(3)^{-1}\left[1-\frac{1}{135}+\frac{\frac{6}{25}}{2}\left(\frac{1}{729}\right)+-----\right]$$

$$\frac{1}{5\sqrt{252}}=\frac{1}{3}\left[1-\frac{1}{135}+\frac{3}{25}\left(\frac{1}{729}\right)+-----\right]$$

$$=\left[\frac{1}{3}-\frac{1}{405}+\frac{1}{18225}+-----\right]$$ Ans.

60. Use Binomial Series, find $(1.03)^{\frac{1}{3}}$ upto three decimal places. (3 times)

Sol: $(1.03)^{\frac{1}{3}}=(1+0.03)^{\frac{1}{3}}$

$$=\left[1+\left(\frac{1}{3}\right)(0.03)+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}(0.03)^2+-----\right]$$

$$=\left[1+0.01+\frac{-\frac{2}{9}}{2}(0.0009)+------\right]$$

$$(1.03)^{\frac{1}{3}}=[1+0.01-0.0001+-----]$$ Ans.

61. Show the formula is true for n = 1, 2

Sol: $1^3+3^3+5^3+---------\mp(2n-1)^3=n^2[2n^2-1]$

$1^3+3^3+5^3+---------\mp(2n-1)^3=n^2[2n^2-1]$

Put $n=1$

$1^3=(1)^2[2(1)^2-1]$

$1=1[2-1]$

$1=1$

⇨ Condition is true for n = 1

Now put $n=2$

$1^3+3^3=(2)^2[2(2)^2-1]$

$1+27=4[2(4)-1]$

$=4[8-1]$

$=4[7]$

$28=28$

⇨ Now condition is true for n = 2

So formula is true for n = 1, 2.

2019

62: Show that $\frac{n^3+2n}{3}$ represents an integer for $n=2,3$

Sol: Given $\frac{n^3+2n}{3}$

for $n=2$

$$=\frac{(2)^3+2(2)}{3}=\frac{8+4}{3}=\frac{12}{3}=4$$

for $n=3$

$$=\frac{(3)^3+2(3)}{3}=\frac{27+6}{3}=\frac{33}{3}=11$$

So $\frac{n^3+2n}{3}$ represents an integer.

63: Expand $\left(1-\frac{3}{2}x\right)^{-2}$ upto 4 terms

Sol: Given $\left(1-\frac{3}{2}x\right)^{-2}$

We know that Binomial Series

$$(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\ldots$$

$$\left(1-\frac{3}{2}x\right)^{-2} = 1+(-x)\left(\frac{-3}{2}x\right)+\frac{-2(-2-1)}{2!}\left(\frac{-3}{2}x\right)^2+\frac{-2(-2-1)(-2-2)}{3!}$$

$$=1+3x+\frac{(-2)(-3)}{2.1}\frac{9x^2}{4}+\frac{(-2)(-3)(-4)}{3.2.1}\left(\frac{-27x^3}{8}\right)+\ldots$$

$$=1+3x+\frac{27}{4}x^2+\frac{27x^3}{2}+\ldots$$

64: Evaluate $\sqrt[3]{30}$ correct to three places of decimal. **(2 times)**

Sol: Given $\sqrt[3]{30}=(30)^{1/3}=(27+3)^{1/3}$

$$=\left[27\left(1+\frac{3}{27}\right)\right]^{1/3}=(27)^{1/3}\left(1+\frac{1}{9}\right)^{1/3}$$

$$=(3^3)^{1/3}\left(1+\frac{1}{9}\right)^{1/3}$$

$$=3\left(1+\frac{1}{9}\right)^{1/3}$$

We know that Binomial Series

$$(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\ldots \text{ for } |x|<1$$

$$=3\left[1+\frac{1}{3}\left(\frac{1}{9}\right)+\right]\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}\left(\frac{1}{9}\right)^2+\ldots$$

$$=3\left[1+\frac{1}{27}-\frac{1}{9}\left(\frac{1}{81}\right)+\ldots\right]$$

$$=3[1+0.03704-0.001372+\ldots]$$

$$=3(1.035668)$$

$$=3.107004$$

Hence $\sqrt[3]{30}\approx 3.107$

65: Use binomial theorem to expand $\left(a-\sqrt{2}x\right)^4$

Sol: Given $\left(a-\sqrt{2}x\right)^4$

We know that Binomial theorem

$$(a+x)^n=\binom{n}{0}a^nx^0+\binom{n}{1}a^{n-1}x^1+\ldots+\binom{n}{n}a^0x^n$$

$$\left(a-\sqrt{2}x\right)^4=\binom{4}{0}a^4\left(-\sqrt{2}x\right)^0+\binom{4}{1}a^3\left(-\sqrt{2}x\right)^1+\binom{4}{2}a^2\left(-\sqrt{2}x\right)^2$$

$$+\binom{4}{3}a^1\left(-\sqrt{2}x\right)^3+\binom{4}{4}a^0\left(-\sqrt{2}x\right)^4$$

$$=1\left(a^4\right).1+4a^3\left(-\sqrt{2}x\right)+6a^2\left(2x^2\right)+4a\left(-2\sqrt{2}x^3\right)+1(1)\left(4x^4\right)$$

$$\left(a-\sqrt{2}x\right)^4=a^4-4\sqrt{2}\,a^3x+12a^2x^2-8\sqrt{2}ax^3+4x^4$$

66: Prove that $n!>2^n-1$ is true for $n=5, n=6$

Sol: Given $n!>2^n-1$

for $n=5$

$\Rightarrow 5!>(2)^5-1$

$120>32-1$

$120>3$ true.

for $n=6$

$\Rightarrow 6!>(2)^6-1$

$720>64-1$

$720>63$ true

Hence $n!>2^n-1$ is true for $n=5,6$.

67: If $1+2+4+...+2^{n-1}=2^n-1$, then check the statement for $n=2$ and $n=3$ is either true or false.

Sol: Let $S(n)$ be the given statement,

then

$S(n):1+2+4+......+2^{n-1}=2^n-1 \rightarrow (i)$

For $n=2$

L.H.S = $1+2=3$

R.H.S = $(2)^2-1=4-1=3$

L.H.S= R.H.S

For $n=3$

L.H.S = 1+2+4=7

R.H.S = $(2)^3-1=8-1=7$

L.H.S.= R.H.S

Hence statement is true for $n=2,3$

68: Expand $(1+x)^{-1/3}$ upto 4 terms **(2 times)**

Sol: Given $(1+x)^{-1/3}$

We know that Binomial series

$$(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+...$$

$$\therefore (1+x)^{-1/3}=1+\left(\frac{-1}{3}x\right)+\frac{\left(-1/3\right)\left(\frac{-1}{3}-1\right)}{2!}x^2+\frac{\left(-1/3\right)\left(-1/3-1\right)\left(-1/3-2\right)}{3!}x^3+...$$

$$=1-\frac{1}{3}x+\frac{\left(-1/3\right)\left(-4/3\right)}{2.1}x^2+\frac{\left(-1/3\right)\left(-4/3\right)\left(-7/3\right)}{3.2.1}x3+...$$

$$= 1 - \frac{1}{3}x + \frac{4}{9}\left(\frac{1}{2}\right)x^2 = \frac{28}{27}\left(\frac{1}{6}\right)x^3 + \ldots$$

$$= 1 - \frac{1}{3}x + \frac{2}{9}x^2 - \frac{14}{81}x^3 + \ldots$$

69: **Expand $(4-3x)^{1/2}$ upto three terms** **(6 times)**

Sol: Given $(4-3x)^{1/2} = \left[4\left(1-\frac{3}{4}x\right)\right]^{1/2}$

$$= (4)^{\frac{1}{2}}\left[1-\frac{3}{4}x\right]^{1/2}$$

$$= (2^2)^{1/2}\left[1-\frac{3}{4}x\right]^{1/2}$$

$$= 2\left[1-\frac{3}{4}x\right]^{1/2}$$

We know that binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots \text{ for } |x|<1$$

$$= 2\left[1 + \frac{1}{2}\left(\frac{-3}{4}x\right) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(\frac{-3}{4}x\right)^2 + \ldots\right]$$

$$= 2\left[1 - \frac{3}{8}x + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2.1}\frac{9}{16}x^2 + \ldots\right]$$

$$= 2\left[1 - \frac{3}{8}x - \frac{9}{128}x^2 + \ldots\right]$$

$$= 2 - \frac{3}{4}x - \frac{9}{64}x^2 + \ldots$$

70: **Prove the formula for $n=1$ and $n=2, 1+4+7+\ldots\ldots(3n-2) = \frac{n(3n-1)}{2}$**

Sol: Let $S(n)$ be the given statement

then

$$S(n): 1+4+7+\ldots\ldots+(3n-2) = \frac{n(3n-1)}{2}$$

For $n=1$

L.H.S = 1

R.H.S = $\frac{1(3(1)-1)}{2} = \frac{1(3-1)}{2} = \frac{2}{2} = 1$

L.H.S =R.H.S

For $n=2$

L.H.S = 1+4=5

R.H.S = $\frac{2(3(2)-1)}{2} = (6-1) = 5$

L.H.S = R.H.S

Hence statement is true for $n=1,2$

71: **Prove that** $\frac{(8\times10^n)-2}{6}$ **is an integer for** $n=1$ **and** $n=2$

Sol: Given $\frac{(8\times10^n)-2}{6}$

for $n=1$

$=\frac{[8\times(10)^1]-2}{6}=\frac{(8\times10)-2}{6}$

$=\frac{80-2}{6}=\frac{78}{6}=13$

for $n=2$

$=\frac{[8\times(10)^2]-2}{6}=\frac{(8\times100)-2}{6}$

$=\frac{800-2}{6}=\frac{798}{6}=133$

Hence statement is an integer for $n=1$ and $n=2$

72: If $1+2+4+...+2^{n-1}=2^n-1$, then verify for $n=3$

Sol: Let $S(n)$ be the given statement,

then

$S(n): 1+2+4+...+2^{n-1}=2^n-1$

for $n=3$

L.H.S = 1+2+4=7

R.H.S = $(2)^3-1=8-1=7$

L.H.S = R.H.S

Hence statement is true for $n=3$

73: **Expand** $(1-x)^{-1/2}$ **upto 3 terms**

Sol: Given $(1-x)^{-1/2}$

We know that binomial series

$(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+...\ |x|<1$

$(1-x)^{-1/2}=1+\left(\frac{-1}{2}\right)(-x)+\frac{(-1/2)\left(\frac{-1}{2}-1\right)}{2!}(-x)^2+...$

$=1+\frac{x}{2}+\frac{(-1/2)(-3/2)}{2.1}x^2+...$

$=1+\frac{x}{2}+\frac{3}{8}x^2+.......$

74: **Using Binomial Series. Find the value of** $\sqrt[5]{31}$

Sol: Given $\sqrt[5]{31}=(31)^{1/5}$

$=(32-1)^{1/5}$

$=\left[32\left(1-1/32\right)\right]^{1/5}$

$=(32)^{1/5}\left(1-\frac{1}{32}\right)^{1/5}$

$= (2^5)^{1/5}\left(1-\frac{1}{32}\right)^{1/5}$

$= 2\left(1-\frac{1}{32}\right)^{1/5}$

We know that binomial series

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots \quad |x|<1$

$= 2\left[1+\left(\frac{1}{5}\right)\left(\frac{-1}{32}\right)+\frac{\left(\frac{1}{5}\right)\left(\frac{1}{5}-1\right)}{2!}\left(\frac{-1}{32}\right)^2+\ldots\right]$

$= 2\left[1-\frac{1}{160}+\frac{\frac{1}{5}\left(\frac{-4}{5}\right)}{2.1}\left(\frac{1}{1024}\right)\right]$

$= 2\left[1-\frac{1}{160}-\left(\frac{4}{50}\right)\left(\frac{1}{1024}\right)\right]$

$= 2\left[1-\frac{1}{160}-\frac{1}{12800}\right]$

$= 2(1-0.0063-0.0001)$

$= 2(0.9936) = 1.9872$

$= 1.987$

75: Write the principles of Mathematical induction.

Sol: The principle of mathematical induction as follows

If statement $S(n)$ for each positive integer n is such that

i. S(1) is true i.e S(n) is true for n=1 and

ii. S(k+1) is true whenever S(k) is true for any positive integer k, then S(n) is true for all positive integers.

76: Prove that the result $3^n < n!$ is true for $n = 7,8$

Sol: Given $3^n < n!$

For $n = 7$

$(3)^7 < 7!$

$2187 < 5040$ (is ture)

for $n = 8$

$(3)^8 < 8!$

$6561 < 40320$ is (true)

Hence statement is true for $n = 7,8$

77: Use binomial theorem to calculate $(21)^5$

Sol: Given $(21)^5$

$= (20+1)^5$

$= \binom{5}{0}(20)^5(1)^0 + \binom{5}{1}(20)^4(1)^1 + \binom{5}{2}(20)^3(1)^2$

$+ \binom{5}{3}(20)^2(1)^3 + \binom{5}{4}(20)^1(1)^4 + \binom{5}{5}(20)^0(1)^5$

$= 1(3200000)1 + 5(1600000)1 + 10(8000)1 + 10(400)1 + 5(20)1 + 1(1)1$

$= 3200000 + 8000000 + 80000 + 4000 + 100 + 1$
$= 11284101$

78: Calculate $(2.02)^4$ by means of binomial Theorem.

Sol: Given $(2.02)^4$

$= (2 + 0.02)^4$

$= \binom{4}{0}(2)^4(0.02)^0 + \binom{4}{1}(2)^3(0.02)^1 + \binom{4}{2}(2)^2(0.02)^2$
$+ \binom{4}{3}(2)^1(0.02)^3 + \binom{4}{4}(2)^0(0.02)^4$

$= 1(16)(1) + 4(8)(0.02) + 6(4)(0.0004) + 4(2)(0.000008) + 1(1)(0.00000016)$
$= 16 + 0.64 + 0.0096 + 0.000064 + 0.00000016$
$= 16.64966416$

2021

79- Show that the inequality $4^n > 3^n + 4$ is true, for integral values of $n \geq 2$.

Sol: $S(n): \quad 4^n > 3^n + 4 \quad$ *for all* $n \geq 2$

For n = 2

$S(2); \quad 4^2 > 3^2 + 4$

$S(2): \quad 16 > 9 + 4$

$S(2): \quad 16 > 13$

Which is true, so condition is satisfied.

II Let the statement be true for any $n = k(\geq 2) \in Z$ that is

$4^k > 3^k + 4 ....(A)$

Multiplying both sides of inequality (A) by 4

We get,

$\Rightarrow 4.4^k > 4(3^k + 4)$

$\Rightarrow 4^{k+1} > (3+1)3^k + 16$

$\Rightarrow 4^{k+1} > 3^{k+1} + 4 + 3^k + 12$

$\Rightarrow 4^{k+1} > 3^{k+1} + 4 \; ....(B) \; \because 3^k + 12 > 0$

The inequality (B), Satisfies the conditions(2)

Hence S(n) is true for all integers $n \geq 2$

80- Calculate $(9.98)^4$ by means of binomial theorem. **(3 Times)**

Sol: $(9.98)^4 = (10 - 0.02)^4$

$= \binom{4}{0}(10^4)(-0.02)^0 + \binom{4}{1}(10)^3(-0.02)^1 + \binom{4}{2}(10)^2(-0.02)^2$

$+ \binom{4}{3}(10)^1(-0.02)^3 + \binom{4}{4}(10)^0(-0.02)^4$

$= 1(1000)(1) + 4(1000)(-0.02) + 6(100)(0.0004) + 4(10)(-0.000008) + 1(1)(0.00000016)$
$= 10000 - 80 + 0.24 - 0.0032 + 0.00000016$
$= 9920.23968016$

81- Verify the statement $2 + 6 + 18 + \text{———} + 2 \times 3^{n-1} = 3^n - 1$ for n = 1 , 2

Sol: Given statement

$S(n): 2+6+18+\ldots\ldots 2\times 3^{n-1} = 3^n - 1 \quad for\ n=1,2$

For n = 1

$S(1): \ 2 = 3^1 - 1$

$2 = 2$

Which is true for n = 1

For n = 2

$S(2): \ 2+6 = 3^2 - 1$

$8 = 9 - 1$

$8 = 8$

Hence $S(n)$ is true for n = 1,2

82- Use mathematical induction to prove that 1 + 5 + 9 + ... + (4n – 3) = n(2n – 1) is ture for n = 1 and n= 2 (3 Times)

Sol: Given statements

$S(n): 1+5+9+\ldots\ldots(4n-3) = n(2n-1) \quad for\ n=1,2$

For n = 1

$S(1): \ 1 = 1(2(1)-1)$

$1 = 1(2-1)$

$1 = 1(1)$

$1 = 1$

$S(n)$ is true for n = 1

For n = 2

$S(2): \ 1+5 = 2[2(2)-1]$

$6 = 2(4-1)$

$6 = 2(3)$

$6 = 6$

Which is also true for n = 2

Hence $S(n)$ is true for n = 1 and n = 2

83- Determine the middle term of the expansion $\left(\frac{1}{x} - \frac{x^2}{2}\right)^{12}$ (2 Times)

Sol: Given expansion $\left(\frac{1}{x} - \frac{x^2}{2}\right)^{12}$

Here n = 12 is even then

Middle term $= \left(\frac{n}{2}+1\right) th\ term$

$= \left(\frac{12}{2}+1\right) th\ term$

$= (6+1) th\ term$

$=$ 7th term

Here $a=\frac{1}{x},\ b=\frac{-x^2}{2},n=12$

$r+1=7 \Rightarrow r=6$

We know that

$$T_{r+1}=\binom{n}{r}a^{n-r}b^r$$

$$T_{6+1}=\binom{10}{6}\left(\frac{1}{x}\right)^{12-6}\left(\frac{-x^2}{2}\right)^6$$

$$T_7=924\left(\frac{1}{x}\right)^6\frac{x^{12}}{64}$$

$$=\frac{924}{64}x^{-6}.x^{12}$$

$$=\frac{231}{16}x^{12-6}$$

$$=\frac{231}{16}x^6$$

84- Expand by using the binomial theorem $(a + 2b)^5$ (3 Times)

Sol: $(a+2b)^5$

$$=\binom{5}{0}(a)^5(2b)^0+\binom{5}{1}(a)^4(2b)^1+\binom{5}{2}(a)^3(2b)^2+\binom{5}{3}(a)^2(2b)^3+\binom{5}{4}(a)^1(2b)^5$$

$$+\binom{5}{5}(a)^0(2b)^5$$

$$=1(a^5)(1)+5(a^4)(2b)+10(a^3)(4b^2)+10(a^2)(8b^3)+5(a)(16b^5)+1(1)(32b^5)$$

$$=a^5+10a^4b+40a^3b^2+80a^2b^3+80ab^4+32b^5$$

LONG QUESTION'S OF CHAPTER-8 IN ALL PUNJAB BOARDS 2011-2021

Topic I: Mathematical Induction:

1. Prove that: x + y is a factor of $x^n - y^n$, $(x \neq y)$ for all $n \in N$.
2. Use mathematical induction and prove $2 + 4 + 6 + \dots\dots + 2n = n\ (n + 1)$ for all positive integral values of n.
3. Use the mathematical induction to prove that $1 + 4 + 7 + - - - + (3n - 2) = \frac{n\,(3n-1)}{2}$
4. Use mathematical Induction to prove the following formula for every positive Integer n $1+\frac{1}{2}+\frac{1}{4}+\dots\dots\dots+\frac{1}{2^{n-1}}=2\left[1-\frac{1}{2^n}\right]$ **(3 times)**

Topic II: Binomial Theorem with Positive Integers:

5. Find the coefficient of x^5 in the expansion of $\left(x^2-\frac{3}{2x}\right)^{10}$. **(5 times)**
6. Determine the middle term in the expansion of $\left(\frac{1}{x}-\frac{x^2}{2}\right)^{12}$. **(2 times)**
7. Find the term involving x^{-2} in the $\left(x-\frac{2}{x^2}\right)^{13}$.

8. Find the coefficient of x^5 in the expansion of $\left(x^2 - \frac{3}{2x}\right)^{10}$. **(2 times)**

9. Find the term Independent of x of $\left(\sqrt{x} + \frac{1}{2x^2}\right)^{10}$

10. Find the term involving x4 in the expansion of $(3 - 2x)^7$ **(4 times)**

11. Determine the middle term in $\left(\frac{3}{2}x - \frac{1}{3x}\right)^{11}$

12. Find the co-efficient of the term involving x^{-1} in the expansion of $\left(\frac{3x}{2} - \frac{1}{3x}\right)^{11}$. **(2 times)**

Topic III: Binomial Expression:

13. If $y = \frac{2}{5} + \frac{1.3}{2!}\left(\frac{2}{5}\right)^2 + \frac{1.3.5}{5!}\left(\frac{2}{5}\right)^3 + ---$ then prove that $y^2 + 2y - 4 = 0$ **(2 times)**

14. If $y = \frac{1}{3} + \frac{1.3}{2!}\left(\frac{1}{3}\right)^2 + \frac{1.3.5}{3!}\left(\frac{1}{3}\right)^3 + ------$then prove that $y^2 + 2y - 2 = 0$ **(4 times)**

15. If x is very nearly equal 1 , then prove that $px^p - qx^p = (p - q)x^{p-q}$

16. If $2y = \frac{1}{2^2} + \frac{1.3}{2!}.\frac{1}{2^4} + \frac{1.3.5}{3!}.\frac{1}{2^6} + ---------------$then prove that $4y^2 + 4y - 1 = 0$

17. If x is so small that its square and higher powers can be neglected then show that $\frac{(1+x)^{\frac{1}{2}}(4-3x)^{\frac{3}{2}}}{(8+5x)^{\frac{1}{3}}} = 4\left(1 - \frac{5}{6}x\right)$

18. Use Mathematical induction to prove the formula for every positive integer n. $1 + \frac{1}{2} + \frac{1}{4} + --------- + \frac{1}{2^{n-1}} = 2\left[1 - \frac{1}{2^n}\right]$

19. Use binomial theorem to show that $1 + \frac{1}{4} + \frac{1.3}{4.8} + \frac{1.3.5}{4.8.12} + ... = \sqrt{2}$. **(2 times)**

20. If x is very nearly equal to 1 then prove that $px^p = qx^q = (p-q)\,x^{p+q}$

21. Identify the series: $1 + \frac{1}{3} + \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + ...$ As a binomial expansion and find its sum.

22. Find the term independent of x in the expansion of $\left(x - \frac{2}{x}\right)^{10}$

23. Use mathematical induction to prove that $1^3 + 3^3 + 5^3 + ... + (2n-1)^3 = n^2(2n^2 - 1)$

24. Find 6th term in the expansion of $\left(x^2 - \frac{3}{2x}\right)^{10}$

25. If x is so small that its square and higher powers can be neglected then shot that $\frac{\sqrt{1+2x}}{\sqrt{1-x}} \approx 1 + \frac{3x}{2}$

26. If $y = \frac{5}{5} + \frac{1.3}{2!}\left(\frac{2}{5}\right)^2 + \frac{1.3.5}{3!}\left(\frac{2}{5}\right)^3 + ----$ then prove that $y^2 + 2y - 4 = 0$

27. Use mathematical induction to prove $\binom{3}{3} + \binom{4}{3} + \binom{5}{3} + --- \binom{n+2}{3} = \binom{n+3}{4}$ for every positive integers n.

OBJECTIVE MCQ'S OF CHAPTER-9 IN ALL PUNJAB BOARDS 2011-2021

Topic I: Units of Meaures of Angle:

1. The system in which angle is measured in radians is called: (2 times)
(A) Sexagesimal (B) Circular (C) Mechanical (D) Symmetric

2. In one hour, the hour hand of a clock turns through an angle: (1 time)
(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

3. With usual notation ℓ equal to: (3 times)
(A) r (B) θ (C) $r\theta$ (D) None

4. The sum of all the angles of the triangle is: (2 times)
(A) 90° (B) 180° (C) 270° (D) 360°

5. $\frac{5\pi}{4}$ radian = (8 times)
(A) 360° (B) 335° (C) 270° (D) 225°

6. $\frac{2\pi}{3}$ is equivalent to: (3 times)
(A) 150° (B) 130° (C) 270° (D) 120°

7. 1° equals: (6 times)
(A) $\frac{\pi}{180}$ radian (B) $\frac{180}{\pi}$ radian (C) $\frac{\pi}{90}$ radian (D) $\frac{\pi}{360}$ radian

8. The central angle of an arc of a circle whose length is equal to the radius of the circle is called: (2 times)
(A) Degree (B) Radian (C) Minute (D) Second

9. Area of a sector of circular region of radius r equals: (3 times)
(A) $r^2\theta$ (B) $2r^2\theta$ (C) $\frac{1}{2}r^2\theta$ (D) $\frac{1}{3}r^2\theta$

10. Which of the following is not Quadrant Angle:
(A) $\frac{9}{2}\pi$ (B) 13 π (C) $\frac{4}{3}\pi$ (D) $\frac{\pi}{2}$

11. 1° is approximately equal to:
(A) 1.175 radian (B) 0.0175 degree (C) 0.0175 radians (D) π radian

12. $\frac{\pi}{2}$ radians is an angle:
(A) Acute (B) Obtuse (C) Straight (D) Quadrant

13. One radian is equal to :
(A) 57.296° (B) 57° (C) 56° (D) 0175°

14. In one hour, the angle in radian through by minute hand of a clock is:
(A) $\frac{\overline{\Lambda}}{2}$ (B) $\overline{\Lambda}$ (C) $\frac{3\overline{\Lambda}}{2}$ (D) +2$\overline{\Lambda}$

Topic II: Trignometric Functions:

15. $1 + \cot^2\theta$ = ____________: (3 times)
(A) $\sin^2\theta$ (B) $\cos^2\theta$ (C) $\sec^2\theta$ (D) $\text{Co sec}^2\theta$

16. If $\tan\theta > 0$, $\sin\theta < 0$ then terminal arm of angle lie in: (5 times)
(A) 1 quad (B) II quad (C) III quad (D) IV quad

17. $(1 - \sin^2\theta)(1 + \tan^2\theta)$ = (4 times)
(A) -1 (B) $\cos^2\theta$ (C) $\sin^2\theta$ (D) 1

18. $\cos(-\theta)$ is: (4 times)
(A) -cosθ (B) sinθ (C) sec θ (D) cos θ

19. If sin θ < 0 , cos θ< θ then θ lies in quadrant: (1 time)
(A) I (B) II (C) III (D) IV

20. $\sin^2 3A + \cos^2 3A$ equal to: **(3 times)**

(A) 4 (B) 3 (C) 2 (D) 1

21. $\cos^2 2\theta + \sin^2 2\theta$ is equal to:

(A) 1 (B) zero (C) $\sec^2\theta$ (D) 2

22. $\frac{\sec\theta}{\operatorname{cosec}\theta}$ is equal to:

(A) $\cos\theta$ (B) $\tan\theta$ (C) $\cot\theta$ (D) $\sin\theta$

23. $\cos(-60^\circ) =$

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $-\frac{\sqrt{3}}{2}$

2018

24. If $\sin\theta = \frac{1}{2}$ then θ is equal to:

(a) 30° (b) 45° (c) 60° (d) 90°

25. If $\tan\theta = \frac{8}{15}$ and $\pi \le \theta \le \frac{3\pi}{2}$ then $\cos\theta =$

(a) $\frac{-17}{15}$ (b) $\frac{17}{15}$ (c) $\frac{15}{17}$ (d) $\frac{-15}{17}$

26. An angle in the standard position whose terminal arm lies on the x – axis or on the y – axis is called:

(a) Obtuse Angle (b) Acute Angle (c) Right Angle (d) Quadrant Angle

27. If ΔABC is right angle triangle such that $m\angle\alpha = 90''$, then with usual notations, the true statement is:

(a) $a^2 = b^2 + c^2$ (b) $b^2 = a^2 + c^2$ (c) $c^2 = a^2 + b^2$ (d) All

2019

28. If $\cos\theta = \frac{1}{\sqrt{2}}$, then θ is equal to: **(2 times)**

(a) 30^0 (b) 45^0 (c) 60^0 (d) 90^0

29. The 60^{th} part of 1-degree is called:

(a) Second (b) minute (c) degree (d) Radian

30. $\frac{9\pi}{5}$ *rad* in degree measure is:

(a) 321^0 (b) 322^0 (c) 323^0 (d) 324^0

31. The measure of angle between hands of a watch at 3 O'clock is:

(a) 30^0 (b) 60^0 (c) 90^0 (d) 120^0

32. $\cot^2\theta - \operatorname{cosec}^2\theta =$:

(a) 2 (b) -1 (c) 1 (d) 0

33. The vertex of an angle in standard form is at:

(a) (1, 0) (b) (0, 1) (c) (1, 1) (d) (0, 0)

34. $\sqrt{2}\sin 45^0 + \frac{1}{\sqrt{2}}\csc 45^0 =$: **(2 times)**

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 2

35. If $\sin\theta = \frac{1}{2}$ *and* $\frac{\pi}{2} < \theta < \pi$ then $\cos\theta =$:

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{-\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

36. In anti clock wise direction $\frac{1}{4}$ rotation is equal to

(a) 90^0 (b) 180^0 (c) 270^0 (d) 45^0

2021

37. **60th part of a minute is called.**

(a) Second (b) Minute (c) Degree (d) Hour

38. **What angle is quadrantal angle?**

(a) 120° (b) 270° (c) 60° (d) 45°

39. **The value of** $\tan\theta \; for \; \theta = 30'' \; is$

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

40. **Which of the following is quadrental Angle**

(a) 350° (b) -390° (c) -360° (d) 410°

41. $\frac{-9\pi}{2}$ **coincides with**

(a) OX (b) OY (c) OX′ (d) OY′

42. $\frac{1}{2}$ **Rotation in clock wise direction equal to**

(a) 180° (b) -180° (c) 90° (d) -90°

43. $\sec\theta.\cos ec\theta.\sin\theta.\cos\theta =$

(a) 1 (b) -1 (c) 0 (d) cannot be determined

44. **In a right angled triangle, the side opposite to right angle is called:**

(a) Base (b) Hypotenuse (c) Perpendicular (d) Altitude

45. **Value of** $\sin 7\pi$ **is equal to:**

(a) 1 (b) ½ (c) -1 (d) 0

46. **Angle** $\frac{5\pi}{9}$ **lies in quadrant.**

(a) 1 (b) III (c) II (d) IV

47. **If** $l = 1.5cm, r = 2.5cm$, **then value of** θ **is:**

(a) 3.75 rad (b) $\frac{3}{5} rad$ (c) 0.60 rad (d) $\frac{5}{3} rad$

ANSWERS TO THE MULTIPLE CHOICE QUESTIONS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
B	B	C	B	D	D	A	B	C	C	C	D	A	D	D
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
C	D	D	C	D	A	B	A	A	A	D	A	B	B	D
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
C	B	D	D	B	A	A	B	B	C	D	B	A	B	D
46	47													
C	B													

SHORT QUESTION'S OF CHAPTER-9 IN ALL PUNJAB BOARDS 2011-2021

Topic I: Units of Meaures of Angle:

1. **Convert 54° 45′ into radians.** **(7 times)**

Sol. 54° 45′

$$= 54° + \frac{45^o}{60} = 54° + \left(\frac{3}{4}\right)^0 = \left(\frac{216+3}{4}\right)^0 = \left(\frac{219^0}{4}\right)$$

$$\frac{219^0}{4} = \frac{219}{4} \times \frac{\pi}{180} \text{ radian}$$

$= \frac{219}{720}\pi$ radians

2. Define the Angle and Coterminal angle.

Sol. Two rays with a common starting point form an angle. One of the rays of angle is called initial side and the other as terminal side.

Coterminal Angle: The angles with the same initial and terminal sides are called coterminal angles.

3. Find r when $\ell = 56$ cm, $\theta = 45°$. **(5 times)**

Sol. Given $\ell = 56$ cm $\quad \theta = 45° = \pi/4$

$\because \ell = r\theta \Rightarrow \quad r = \ell/\theta = \frac{56}{\pi/4} = \frac{56\times 4}{\pi} = \frac{224}{3.1415} = 71.3cm$

4. Define radian. **(4 times)**

Sol. Radian is the measure of the angle subtended at the center of the circle by an area whose length is equal to the radius of the circle.

5. Convert 21.56° to the degree, minute, second form.

Sol. 21.256°

$0.256° = (0.256)(1°)$

$= (0.256)(60') = 15.36'$

And $0.36' = (0.36)(1')$

$= (0.36)(60'') = 21.6''$

Therefore, $21.256° = 21° + 0.256°$

$= 21° + 15.36' = 21° + 15' + 0.36' = 21° + 15' + 21.6'' = 21°\ 15'\ 21''$

6. Express the sexagasimal measures of angle in radian 75° 6'30". **(2 times)**

Sol. $75°6'30'' = 75° + \frac{6°}{60} + \frac{30°}{3600} = \left(75 + \frac{1}{10} + \frac{1}{120}\right)^o$

$= \left(\frac{9000+12+1}{120}\right) = \frac{9013°}{120} = \frac{9013}{120}\times\frac{\pi}{180} = \frac{9013\pi}{21600}$ radian

7. Find 'l' when $\theta = 65°20'$; r = 18 mm. **(2 times)**

Sol. $\theta = 65°20' = 65° + \frac{20°}{60°} = 65° + \frac{1°}{3} = \frac{196°}{3} = \frac{196}{3}(.01745)$ rad.

$= 1.14$ rad.

$\ell = r\theta = (18)(1.14) = 20.52$ mm.

8. Convert in radian measure 35°20'. **(2 times)**

Sol. $35°20' = \left(35 + \frac{20}{60}\right)^o = \left(35 + \frac{1}{3}\right)^o = \left(\frac{106}{3}\right)^o$

$= \frac{106}{3}\times\frac{\pi}{180}$ radian

$= \frac{53\pi}{270}$ radian

9. Convert 150° into radians.

Sol. We know

$180° = \pi$ rad.

$1° = \frac{\pi}{180}$ rad.

$150(1°) = 150\left(\frac{\pi}{180}\right)$ rad.

$150° = \frac{5\pi}{6}$ rad.

10. If $l = 5$cm $\theta = \frac{1}{2}$ radians Find r. **(2 times)**

Sol. We know

$l = r\theta$

$5 = r\left(\frac{1}{2}\right)$

$10 = r$

$r = 10$ cm

11. Find θ, when l = 3.2m, r = 2m.

Sol. Given L = 3.2m

We know that

$$\theta = \frac{l}{r}$$

$$\theta = \frac{3.2}{2} \Rightarrow \theta = 1.6$$

12. Convert $18^0\ 6'\ 21''$ to decimal form. **(2 Times)**

Sol Given $18^0\ 6'\ 21''$

$$= 18^0 + 6' + 21''$$

$$= 18^0 + \left(\frac{6}{60}\right)^0 + \left(\frac{21}{3600}\right)^0$$

$$= (18 + 0.1 + 0.005833)^0$$

$$= 18.105833^0$$

13. Find θ, when l =1.5 cm, r = 2.5cm. **(3 times)**

Sol Given Find "θ"

l = 1.5 cm r = 2.5 cm

We know that

$$\theta = \frac{l}{r}$$

$$\theta = \frac{1.5}{2.5} \Rightarrow \theta = 0.6 \text{ radians}$$

14. Convert $120'\ 40''$ into radian measure. **(2 times)**

Sol

$120'\ 40''$ into radian

$$= 120' + 40''$$

$$= \left(\frac{120}{60}\right)^0 + \left(\frac{40}{60\times 60}\right)^0 = 2^0 + \left(\frac{1}{90}\right)^0 = \left(2 + \frac{1}{90}\right)^0$$

$$= \frac{181^0}{90} \qquad \because 1^0 = \frac{\pi}{180} \text{ rad}$$

$$= \frac{181}{90} \times \frac{\pi}{180}$$

$$= \frac{181\pi}{16200} \text{ rad}$$

Topic II: Trignometric Functions:

15. Evaluate $\frac{\tan\pi/3 - \tan\pi/6}{1+\tan\pi/3\tan\pi/6}$ **(3 times)**

Sol. $$\frac{\tan\pi/3 - \tan\pi/6}{1+\tan\pi/3\tan\pi/6} = \frac{\tan 60^o - \tan 30^o}{1+\tan 60^o\tan 30^o} = \frac{\sqrt{3}-\frac{1}{\sqrt{3}}}{1+\sqrt{3}.\frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{3-1}{\sqrt{3}}}{1+1} = \frac{\frac{2}{\sqrt{3}}}{2} = \frac{2}{\sqrt{3}} \times \frac{1}{2} = \frac{1}{\sqrt{3}}$$

16. Prove that: $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$ **(3 times)**

Sol. L.H.S $= (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$

$= \sec^2\theta - \tan^2\theta \qquad \because 1 + \tan^2\theta = \sec^2\theta$

$= 1$, $1 = \sec^2 - \tan^2\theta$

$=$ R.H.S

17. Find the values of remaining trigonometric function if $\sin\theta = \frac{12}{13}$ and terminal arm of the angle is in quadrant 1st .

Sol. $\sin\theta = \frac{12}{13}$

As $\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta + \left(\frac{12}{13}\right)^2 = 1$

$\cos^2\theta = 1 - \frac{144}{169}$

$\cos^2\theta = \frac{169-144}{169} = \frac{25}{169}$

$\cos\theta = +\frac{5}{13}$ $\because \theta$ lies in 1st quad.

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{12/13}{5/13} = \frac{12}{5}$

All trigonometric functions are

$\sin\theta = 12/13$, $\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{13}{12}$

$\cos\theta = 5/13$, $\sec\theta = \frac{1}{\cos\theta} = \frac{13}{5}$

$\tan\theta = 12/5$, $\cot\theta = \frac{1}{\tan\theta} = \frac{5}{12}$ Ans.

18. Prove the identity: $\cot^2\theta - \cos^2\theta = \cos^2\theta\cot^2\theta$.

Sol. L.H.S $\cot^2\theta - \cos^2\theta$

$= \frac{\cos^2\theta}{\sin^2\theta} - \frac{\cos^2\theta}{1} = \frac{\cos^2\theta - \cos^2\theta\sin^2\theta}{\sin^2\theta}$

$= \frac{\cos^2\theta(1-\sin^2\theta)}{\sin^2\theta}$

$= \frac{\cos^2\theta}{\sin^2\theta}\cos^2\theta$ $(\because \cos^2\theta = 1 - \sin^2\theta)$

$= \cot^2\theta.\cos^2\theta$ $(\because \frac{\cos\theta}{\sin\theta} = \cot\theta)$

$=$ R.H.S

19. Discuss the signs of Trigonometric function in III and IV quadrant.

Sol. In III quad $\sin\theta < 0$, $\cos\theta < 0$, $\tan\theta > 0$

cosec < 0, sec < 0, $\cot\theta > 0$

In IV quadrant:

$\sin\theta < 0$, $\cos\theta > 0$, $\tan\theta < 0$

cosec < 0, sec > 0, $\cot\theta < 0$

20. Prove that $\tan\theta + \cot\theta = \text{cosec}\,\theta\sec\theta$

Sol. L.H.S $= \tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$ (Taking L.C.M)

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ $\therefore \sin^2\theta + \cos^2\theta = 1$

$= \frac{1}{\sin\theta\cos\theta}$ $\Rightarrow$ $= \frac{1}{\sin\theta}.\frac{1}{\cos\theta}$

$= \sec\theta\,\text{cosec}\,\theta$

$=$ R.H.S (Hence Proved)

21. Prove that $\sin^2\frac{\pi}{6} + \sin^2\frac{\pi}{3} + \tan^2\frac{\pi}{4} = 2$ **(4 times)**

Sol. $\sin^2\frac{\pi}{6} + \sin^2\frac{\pi}{3} + \tan^2\frac{\pi}{4} = 2$

L.H.S $=$ $\sin^2\frac{\pi}{6} + \sin^2\frac{\pi}{3} + \tan^2\frac{\pi}{4}$

$= \sin^2 30^\circ + \sin^2 60^\circ + \tan^2 45^\circ$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2 = \frac{1}{4} + \frac{3}{4} + 1 = \frac{1+3}{4} + 1.$

$= \frac{4}{4} + 1 = 1 + 1 = 2 = R.H.S$

22. Verify that $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = \sin 30^\circ$ **(2 times)**

Sol. $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = \sin 30^\circ$

L.H.S $= \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

R.H.S $= \sin 30^\circ = \frac{1}{2}$

L.H.S = R.H.S

Hence verified the result.

23. State Fundamental Identities. (any three) **(2 times)**

Sol. (1) $\sin^2\theta + \cos^2\theta = 1$

(2) $1 + \tan^2\theta = \sec^2\theta$

(3) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

24. Prove that $1 + \cot^2\theta = \operatorname{cosec}^2\theta$. **(2 times)**

Sol. In right triangle ABC by Pythagora's theorem.

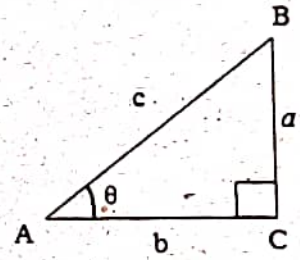
$a^2 + b^2 = c^2$ ______ Eq(1)

Dividing (1) by a^2

$$\frac{a^2}{a^2} + \frac{b^2}{a^2} = \frac{c^2}{a^2}$$

$$1 + \left(\frac{b}{a}\right)^2 = \left(\frac{c}{a}\right)^2$$

$$\Rightarrow \quad 1 + \cot^2\theta = \operatorname{cosec}^2\theta$$



25. Prove that $2\sin 45^\circ + \frac{1}{2}\operatorname{cosec} 45^\circ = \frac{3}{\sqrt{2}}$ **(7 times)**

Sol. L.H.S $= 2\sin 45^\circ + \frac{1}{2}\operatorname{cosec} 45^\circ$

$= 2\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{2}(\sqrt{2})$ $\quad \because \sin 45^\circ = \frac{1}{\sqrt{2}} \Rightarrow \csc 45^\circ = \sqrt{2}$

$= \frac{2}{\sqrt{2}} + \frac{\sqrt{2}}{2} = \frac{4 + (\sqrt{2})^2}{2\sqrt{2}} = \frac{4+2}{2\sqrt{2}}$

$= \frac{6}{2\sqrt{2}} = \frac{3}{\sqrt{2}}$ = R.H.S

26. Prove that $(\sec\theta - \tan\theta)^2 = \frac{1-\sin\theta}{1+\sin\theta}$ **(2 times)**

Sol. L.H.S $= (\sec\theta - \tan\theta)^2$

$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$

$= \frac{(1-\sin\theta)^2}{\cos^2\theta} = \frac{(1-\sin\theta)^2}{1-\sin^2\theta} = \frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)} = \frac{1-\sin\theta}{1+\sin\theta}$ = R.H.S

27. Prove that $\frac{\sin\theta}{1+\cos\theta} + \cot\theta = \operatorname{cosec}\theta$ **(2 times)**

Sol. L.H.S $= \frac{\sin\theta}{1+\cos\theta} + \cot\theta$

$= \frac{\sin\theta}{1+\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos\theta(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$

$= \frac{\sin^2\theta + \cos\theta + \cos^2\theta}{(1+\cos\theta)\sin\theta} = \frac{\sin^2\theta + \cos^2\theta + \cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{1+\cos\theta}{(1+\cos\theta)\sin\theta} = \frac{1}{\sin\theta} = \operatorname{cosec}\theta$ = R.H.S

28. Find x when $\tan^2 45^\circ - \cos^2 60^\circ = x \sin 45^\circ \cos 45^\circ \tan 60^\circ$ **(9 times)**

Sol. $\tan^2 45^\circ - \cos^2 60^\circ = x \sin 45^\circ \cos 45^\circ \tan 60^\circ$

$1 - \left(\frac{1}{2}\right)^2 = x\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)\sqrt{3}$

$1 - \frac{1}{4} = x\left(\frac{1}{\sqrt{2}}\right)^2\sqrt{3}$

$\frac{4-1}{4} = x\frac{1}{2}\sqrt{3}$

$\frac{3}{4} = \frac{\sqrt{3}}{2}x$

Or

$\frac{\sqrt{3}}{2}x = \frac{3}{4}$

$x = \frac{3}{4} \times \frac{2}{\sqrt{3}} = \frac{\sqrt{3}\sqrt{3}}{4} \times \frac{2}{\sqrt{3}} = \frac{\sqrt{3}}{2}$

29. **Prove that $2\cos^2\theta - 1 = 1 - 2\sin^2\theta$** **(3 times)**

Sol. L.H.S = $2\cos^2\theta - 1$

$= 2(1-\sin^2\theta) - 1 = 2 - 2\sin^2\theta - 1 = 1 - 2\sin^2\theta$ = R.H.S

30. **Prove that: $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$** **(3 times)**

Sol. L.H.S = $\cos^4\theta - \sin^4\theta$

$= (\cos^2\theta)^2 - (\sin^2\theta)^2 = (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$

$= 1(\cos^2\theta - \sin^2\theta) = \cos^2\theta - \sin^2\theta$ = R.H.S

31. **Prove that: $\text{Sin}^2\frac{\pi}{6} : \text{Sin}^2\frac{\pi}{4} : \text{Sin}^2\frac{\pi}{3} : \text{Sin}^2\frac{\pi}{2} = 1:2:3:4$** **(4 times)**

Sol. L.H. S = $\text{Sin}^2\frac{\pi}{6} : \text{Sin}^2\frac{\pi}{4} : \text{Sin}^2\frac{\pi}{3} : \text{Sin}^2\frac{\pi}{2}$

$= \left(\frac{1}{2}\right)^2 : \left(\frac{1}{\sqrt{2}}\right)^2 : \left(\frac{\sqrt{3}}{2}\right)^2 : (1)^2 = \frac{1}{4} : \frac{1}{2} : \frac{3}{4} : 1$

$= 4\left(\frac{1}{4}\right) : 4\left(\frac{1}{2}\right) : 4\left(\frac{3}{4}\right) : 4(1) = 1:2:3:4$ = R.H.S

32. **Prove that: Cos2θ = Cos² θ - Sin² θ When θ = 30°, 45°**

Sol. Cos2 θ = Cos² θ - Sin² θ

$\text{Cos}2(30^\circ) = \text{Cos}^2 30^\circ - \text{Sin}^2 30^\circ$ when $\theta = 30^\circ$

$\text{Cos}60^\circ = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

$\frac{1}{2} = \frac{3}{4} - \frac{1}{4}$

$\frac{1}{2} = \frac{3-1}{4}$

$\frac{1}{2} = \frac{2}{4}$

$\frac{1}{2} = \frac{1}{2}$ Proved

Now when $\theta = 45^\circ$

$\text{Cos}2\theta = \text{Cos}^2\theta - \text{Sin}^2\theta$

$\text{Cos}2(45^\circ) = \text{Cos}^2 45^\circ - \text{Sin}^2 45^\circ$

$\text{Cos}90^\circ = \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$

$0 = \frac{1}{2} - \frac{1}{2}$

0 = 0 (Proved)

33. **Prove That: $(\tan\theta + \cot\theta)^2 = \sec^2\theta\,\text{cosec}^2\theta$** **(2 times)**

Sol. L.H.S = $(\tan\theta + \cot\theta)^2$

$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)^2$

L.H.S = $\left(\frac{1}{\sin\theta\cos\theta}\right)$

$= (\text{cosec}\,\theta \,.\, \sec\theta)^2 = \sec^2\theta\,\text{cosec}^2\theta$ = R.H.S

34. **Prove that: $\frac{2\tan\theta}{1+\tan^2\theta} = 2\sin\theta\cos\theta$** **(2 times)**

Sol. L.H.S = $\frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\tan\theta}{\sec^2\theta} = 2\tan\theta\cos^2\theta$

$= 2\frac{\sin\theta}{\cos\theta}\cos\theta \cdot \cos\theta = 2\sin\theta\cos\theta$

35. **Prove that $\frac{Cot^2\theta-1}{1+Cot^2\theta} = 2\,Cos^2\theta-1$** **(2 times)**

Sol. L.H.S $= \frac{Cot^2\theta-1}{1+Cot^2\theta}$

$= \frac{Cot^2\theta-1}{1+Cot^2\theta} = \frac{1}{Cosec^2\theta}(Cot^2\theta - 1) = Sin^2\theta\left(\frac{Cos^2\theta}{Sin^2\theta} - 1\right)$

$= Sin^2\theta\left(\frac{Cos^2\theta - Sin^2\theta}{Sin^2\theta}\right) = Cos^2\theta - Sin^2\theta = Cos^2\theta - (1 - Cos^2\theta)$

$= Cos^2\theta - 1 + Cos^2\theta = 2Cos^2\theta - 1 =$ R.H.S

36. **Prove that $Sec\theta\, Cosec\theta\, Sin\theta\, Cos\theta = 1$**

Sol. L.H.S = $Sec\theta\, Cosec\theta\, Sin\theta\, Cos\theta$

$= \left(\frac{1}{Cos\theta}\right)\left(\frac{1}{Sin\theta}\right) Sin\theta\, Cos\theta = 1 =$ R.H.S

37. **Prove that $\frac{1-Sin\theta}{Cos\theta} = \frac{Cos\theta}{1+Sin\theta}$** **(5 times)**

Sol. L.H.S $= \frac{1-Sin\theta}{Cos\theta}$

$= \frac{(1-Sin\theta)(1+Sin\theta)}{Cos\theta\,(1+Sin\theta)} = \frac{1-Sin^2\theta}{Cos\theta\,(1+Sin\theta)} = \frac{Cos^2\theta}{Cos\theta\,(1+Sin\theta)} = \frac{Cos\theta}{1+Sin\theta} =$ R.H.S

38. **Prove that $Cot^4 + Cot^4 = Cosec^4\theta - Cosec^2\theta$**

Sol. L.H.S = $Cot^4\theta + Cot^2\theta$

$= Cot^2\theta\,(Cot^2\theta + 1) = Cot^2\theta\,(1 + Cot^2\theta) = (Cosec^2\theta - 1)(Cosec^2\theta)$

$= Cosec^4\theta - Cosec^2\theta =$ R.H.S

39. **Prove that: $\frac{1}{1+Sin\theta} + \frac{1}{1-Sin\theta} = 2Sec^2\theta$** **(2 times)**

Sol. $\frac{1}{1+Sin\theta} + \frac{1}{1-Sin\theta} = 2Sec^2\theta$

L.H.S $= \frac{1}{1+Sin\theta} + \frac{1}{1-Sin\theta} = \frac{1-Sin\theta+1+Sin\theta}{(1+Sin\theta)(1-Sin\theta)}$

$= \frac{1+1}{(1)^2-(Sin\theta)^2}$

$= \frac{2}{1-Sin^2\theta}$ $\quad \because 1-Sin^2\theta = Cos^2\theta$

$= \frac{2}{Cos^2\theta} = 2Sec^2\theta =$ R.H.S

2018

40. **Find the radius of the circle in which the arms of central Angle of Measure 1 radian cut off an Arc of Length 35 cm.**

Ans: $r = ?$

$\theta = 1$ rad

$\ell = 35$ cm

Since $\ell = r\theta$

$35 = r \times 1$

$35cm = r$

41. **Prove $\cos^2\theta - \sin^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$**

Sol: R.H.S

$$= \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\cos^2\theta - \frac{\sin^2\theta}{\cos^2\theta}}{\cos^2\theta + \frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta} \qquad \therefore \sin^2\theta + \cos^2\theta = 1$$

$$= \cos^2\theta - \sin^2\theta \qquad = \text{L.H.S}$$

42. **Prove that** $= \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{\cot\theta - 1}{\cot\theta + 1}$

Sol: R.H.S

$$\frac{\cot\theta - 1}{\cot\theta + 1}$$

$$= \frac{\frac{\cos\theta}{\sin\theta} - 1}{\frac{\cos\theta}{\sin\theta} + 1} = \frac{\cos\theta - \sin\theta}{\cos\theta - \sin\theta}$$

= L.H.S

2019

43: **Find the values of all trigonometric functions for** -15π

Sol: $-15\pi = -16\pi + \pi = (-8)2\pi + \pi = \pi, \quad k = -8$

So values of trigonometric functions at -15π and π are same.

Then $Sin(-15\pi) = Sin\pi = 0$

$Cos(-15\pi) = Cos\pi = -1$

$\tan(-15\pi) = \tan\pi = 0$

$Cot(-15\pi) = Cot\pi = \frac{1}{\tan\pi} = \frac{1}{0} = \infty$

$Sec(-15\pi) = Sec\pi = -1$

$Cosec(-15\pi) = Cosec\pi = \frac{1}{Sin\pi} = \frac{1}{0} = \infty$

44: **Convert** $\frac{9\pi}{5}$ **to sexagesimal system.**

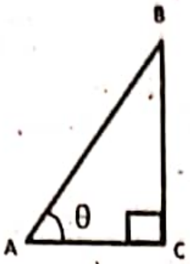
Sol: $\frac{9\pi}{5}$

$$= \frac{9\pi}{5} \times \frac{180}{\pi} \text{degree} \qquad \because 1\ red = \frac{180}{\pi}$$

$= 324$ degree $= 324^0$

45: Define "right angled triangle".

Sol: **Definition:** A triangle which has 90^0 angle is called right angled triangle.
Triangle has six elements with three sides a, b, c and three angles α, β, γ.



46: What is the length of the arc intercepted on a circle of radius 14cms by the arms of a central angle of 45^0?

Sol: Given

$r = 14cm \quad , \quad l = ?$

$\theta = 45^0$

$= 45 \times \frac{\pi}{180} \; rad.$

$\because \pi = 3.14$

$= 0.7854 \quad rad.$

We know that

$l = r\theta$

$= 14(0.7854) = 10.99cm$

47: Find the values of $Sin\theta$ and $Cos\theta$ when $\tan\theta = \frac{-1}{3}$ and the terminal arm of the anlge is in quad II.

Sol: Given $\tan\theta = \frac{-1}{3}$ *and* $\theta \in II \; Quad.$

As

$1 + \tan^2\theta = \sec^2\theta$

$1 + \left(\frac{-1}{3}\right)^2 = \sec^2\theta$

$1 + \frac{1}{9} = \sec^2\theta$

$\frac{10}{9} = \sec^2\theta$

Taking square root on both sid

$\Rightarrow \quad \pm\frac{\sqrt{10}}{3} = \sec\theta$

$\because \quad \theta \in II \; Quad.$

$\frac{-\sqrt{10}}{3} = \sec\theta$

Taking reciprocal on both sides

$\frac{-3}{\sqrt{10}} = \cos\theta$

Now

$$Sin^2\theta + Cos^2\theta = 1$$

$$Sin^2\theta = 1 - Cos^2\theta$$

$$Sin^2\theta = 1 - \left(\frac{-3}{\sqrt{10}}\right)^2$$

$$Sin^2\theta = 1 - \frac{9}{10}$$

$$Sin^2\theta = \frac{1}{10}$$

Taking square root on both si

$$Sin\theta = \pm\frac{1}{\sqrt{10}}$$

$\because \quad \theta \in II\ Quad.$

$$Sin\theta = \frac{-1}{\sqrt{10}}$$

48: Verify the result, $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$ for $\theta = 30^0$

Sol: L.H.S = $\tan 2\theta$ $\quad \because \theta = 30^0$

$= \tan 2(30^0)$

$= \tan 60^0$ $\quad \because \tan 60^0 = \sqrt{3}$

$= \sqrt{3}$

R.H.S $= \dfrac{2\tan\theta}{1-\tan^2\theta}$ $\quad \because \theta = 30^0$

$= \dfrac{2\tan 30^0}{1-(\tan 30^0)^2} = \dfrac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$ $\quad \because \tan 30^0 = \dfrac{1}{\sqrt{3}}$

$= \dfrac{2/\sqrt{3}}{1-\frac{1}{3}} = \dfrac{2/\sqrt{3}}{\frac{3-1}{3}} = \dfrac{2/\sqrt{3}}{2/3} = \dfrac{2}{\sqrt{3}} \times \dfrac{3}{2}$

$= \dfrac{3}{\sqrt{3}}$ $\quad x^1 = \sqrt{x}.\sqrt{x}$

$= \dfrac{\sqrt{3}.\sqrt{3}}{\sqrt{3}} = \sqrt{3}$ $\quad$ Hence L.H.S = R.H.S

49: If $Sin\theta = \dfrac{-1}{\sqrt{2}}$, terminal arm of θ is not in III Quadrant, find $\tan\theta$ **(2 times)**

Sol: $Sin\theta = \dfrac{-1}{\sqrt{2}}$ $\quad \because \theta$ is not in III Quad.

$\theta \notin III\ Quad.$

$\Rightarrow \quad \theta \in IV\ Quad.$

$\because \quad Sin^2\theta + Cos^2\theta = 1$

$Cos^2\theta = 1 - Sin^2\theta$

$$Cos^2\theta = 1-\left(\frac{-1}{\sqrt{2}}\right)^2$$

$$Cos^2\theta = 1-\frac{1}{2}$$

$$Cos^2\theta = \frac{2-1}{2}$$

$$Cos^2\theta = \frac{1}{2}$$

Taking square root on both sides

$$Cos\theta = \pm\frac{1}{\sqrt{2}}$$

Now

$$Tan\theta = \frac{Sin\theta}{Cos\theta}$$

$$Tan\theta = \frac{-1/\sqrt{2}}{1/\sqrt{2}}$$

$$Tan\theta = -1$$

50: Verify: $Cos\theta + \tan\theta Sin\theta = Sec\theta$

Sol: L.H.S $= Cos\theta + \tan\theta Sin\theta$

$$= Cos\theta + \frac{Sin\theta}{Cos\theta}Sin\theta$$

$$= Cos\theta + \frac{Sin^2\theta}{Cos\theta} \qquad \because Cos^2\theta + Sin^2\theta = 1$$

$$= \frac{Cos^2\theta + Sin^2\theta}{Cos\theta}$$

$$= \frac{1}{Cos\theta}$$

$$= Sec\theta$$

$$= R.H.S$$

L.H.S = R.H.S

51: If $Cos\theta = \frac{9}{41}$ and terminal arm of the angle is in quadrant IV then find the values of $\tan\theta, Sin\theta$

Sol: Given $Cos\theta = \frac{9}{41}$ $\qquad \because \theta$ is in IV Quad.

$\because$

$$Cos^2\theta + Sin^2\theta = 1$$

$$Sin^2\theta = 1 - Cos^2\theta$$

$$Sin^2\theta = 1-\left(\frac{9}{41}\right)^2$$

$$Sin^2\theta = 1 - \frac{81}{1681}$$

$$Sin^2\theta = \frac{1681-81}{1681}$$

$$Sin^2\theta = \frac{1600}{1681}$$

$$\Rightarrow \quad Sin\theta = \pm\frac{40}{41}$$

$$\because \quad \theta \in IV\ Quad$$

$$Sin\theta = \frac{-40}{41}$$

Now

$$Tan\theta = \frac{Sin\theta}{Cos\theta}$$

$$Tan\theta = \frac{-44/41}{9/41}$$

$$Tan\theta = \frac{-40}{9}$$

52: **Prove that** $\tan\theta + Cot\theta = Co\sec\theta Sec\theta$

Sol: L.H.S = $\tan\theta + Cot\theta$

$$= \frac{Sin\theta}{Cos\theta} + \frac{Cos\theta}{Sin\theta} = \frac{Sin^2\theta + Cos^2\theta}{Sin\theta Cos\theta} \qquad \because Sin^2\theta + Cos^2\theta = 1$$

$$= \frac{1}{Sin\theta Cos\theta}$$

$$= \left(\frac{1}{Sin\theta}\right)\left(\frac{1}{Cos\theta}\right) \qquad = Co\sec\theta Sec\theta$$

= R.H.S

So L.H.S = R.H.S

53: **Convert** $154^0 20''$ **to radian measure**

Sol: $154^0 20'' = \left(154 + \frac{20}{3600}\right)^0$

$$= \left(154 + \frac{1}{180}\right)^0 \qquad = \left(\frac{27721}{180}\right)^0$$

$$= \frac{27721}{180} \times 1^0 \quad = \frac{27721}{180} \times \frac{\pi}{180}\ rad$$

$$= \frac{27721}{32400}\pi\ rad$$

54: **Evaluate** $\dfrac{1-\tan^2 \pi/3}{1+\tan^2 \pi/3}$ **(4 times)**

Sol: $\dfrac{1-\tan^2 \pi/3}{1+\tan^2 \pi/3} = \dfrac{1-(\sqrt{3})^2}{1+(\sqrt{3})^2} \qquad \because \tan\frac{\pi}{3} = \tan 60^0 = \sqrt{3}$

$= \frac{1-3}{1+3} \qquad = -\frac{2}{4} = -\frac{1}{2}$

2021

55- **Prove that** $\sec^2\theta - \operatorname{cosec}^2\theta = \tan^2\theta - \cot^2\theta$ **(2 Times)**

Sol: L.H.S $= \sec^2\theta - \operatorname{cosec}^2\theta$

$= (1+\tan^2\theta) - (1+\cot^2\theta) \qquad \because 1+\tan^2\theta = \sec^2\theta$

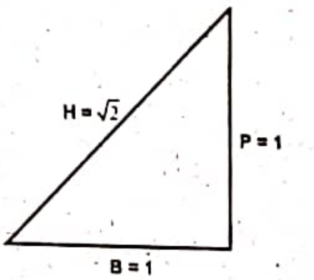
$= 1+\tan^2\theta - 1 - \cot^2\theta \qquad \because 1+\cot^2\theta = \operatorname{cosec}^2\theta$

$= \tan^2\theta - \cot^2\theta = R.H.S$

Hence L.H.S = R.H.S

56- **If** $\sin\theta = -\frac{1}{\sqrt{2}}$ (θ) **is in 3rd quadrant. Find the value of** $\cot\theta$

Sol: $\sin\theta = -\frac{1}{\sqrt{2}}$ and θ is in 3rd quadrant.



By Pythagoras theorem

$H^2 = B^2 + P^2$

$(\sqrt{2})^2 = (1)^2 + B^2$

$2 = 1 + B^2$

$2 - 1 = B^2$

$B = 1$

Since terminal of the angle θ lies in III-Quad

So, $\cot\theta = \frac{B}{p}$

$\cot\theta = \frac{1}{1}$

$\cot\theta = 1$

57- **Verify sin2** θ **= 2sin** θ **cos** θ **, when** θ **= 30° , 45°**

Sol: $\sin 2\theta = 2\sin\theta\cos\theta$

When $\theta = 30^o$

So, $\sin 2(30^o) = 2\sin 30^o \cos 30^o$

$\sin 60^o = 2\left(\frac{1}{2}\right)\frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ (verified)

Now When $\theta = 45^o$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\sin 2(45^o) = 2\sin 45^o \cos 45^o$

$$\sin 90^\circ = 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$1 = \frac{2}{(\sqrt{2})^2}$$

$$1 = \frac{2}{2}$$

$1 = 1$ (verified)

58- **Prove that** $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta, \textit{where}\,\theta$ **is not an odd multiple of** $\dfrac{\pi}{2}$

(2 Times)

Sol: L.H.S $= \sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}$

$$= \sqrt{\frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}} \quad (\textit{rationalizing})$$

$$= \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}} \quad \because a^2-b^2=(a-b)(a+b)$$

$$= \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$

$$= \sqrt{\left(\frac{1-\sin\theta}{\cos\theta}\right)^2}$$

$$= \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta - \tan\theta = R.H.S$$

Hence L.H.S = R.H.S

59- **Find the value of** $\sin\theta$ **and** $\cos\theta$ **if** $\theta = \dfrac{-9\pi}{2}$.

Sol; As we know that $\theta + 2k\pi = \theta \quad ,k \in z$

So, $\dfrac{-9\pi}{2} = -810^\circ$

$$= 270^\circ + (-3)(360^\circ) \quad \because k = -3$$

$$= 270^\circ$$

Now,

$$\sin\left(-\frac{9\pi}{2}\right) = \sin(270^\circ) = -1$$

$$\cos\left(-\frac{9\pi}{2}\right) = \cos(270^\circ) = 0$$

60- **In which quadrant the terminal arms of the angle lie when sec** $\theta < 0$ **and sin** θ **<0.**

Sol: $\sec\theta < 0 \quad \textit{and} \sin\theta < 0$

$\{II, III\} \cap \{III, IV\} = \{III\}$

Hence terminal arm of angle θ lies in III-Quad

61- **Prove that $\sec^2 A + \operatorname{cosec}^2 A = \sec^2 A . \operatorname{cosec}^2 A$, where $A \neq \frac{n\pi}{2}, n \in Z$ (2 Times)**

Sol: L.H.S $= \sec^2 A + \operatorname{cosec}^2 A$

$= \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A}$

$= \frac{\sin^2 A + \cos^2 A}{\cos^2 A \sin^2 A}$

$= \frac{1}{\cos^2 A \sin^2 A}$ $\because \sin^2 A + \cos^2 A = 1$

$= \frac{1}{\cos^2 A} \cdot \frac{1}{\sin^2 A}$

$= \sec^2 A . \operatorname{cosec}^2 A = R.H.S$

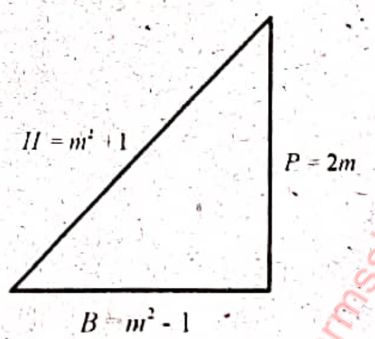
Hence L.H.S = R.H.S

62- **If $\operatorname{cosec}\theta = \frac{m^2+1}{2m}$ $0 < \theta < \frac{\pi}{2}$. Find the value of $\sec\theta$**

Sol: $\operatorname{cosec}\theta = \frac{m^2+1}{2m}$ and $0 < \theta < \frac{\pi}{2}$

By Pythagoras theorem

$H^2 = P^2 + B^2$



$(m^2+1)^2 = (2m)^2 + B^2$

$m^4 + 2m^2 + 1 = 4m^2 + B^2$

$m^4 + 2m^2 + 1 - 4m^2 = B^2$

$m^4 - 2m^2 + 1 = B^2$

$(m^2-1)^2 = B^2$

$m^2 - 1 = B$

$\Rightarrow B = m^2 - 1$

Since θ lies in I-Quad

So, $\sec\theta = \frac{H}{B}$

$\sec\theta = \frac{m^2+1}{m^2-1}$

63- **Find l, when $\theta = \pi$ radians, $r = 6cm$**

Given

$\theta = \pi\ rad$

$r = 6cm$

We know that

$l = r\theta$

$= 6\times\pi = 6\times 3.1416 \quad \because \pi = 3.1416 \quad = 18.8cm$

LONG QUESTION'S OF CHAPTER-9 IN ALL PUNJAB BOARDS 2011-2021

Trignometric Functions:

1. If cot θ = 5/2 & terminal arm of angle is 1st quadrant, find value $\frac{3\,Sin\theta+4Cos\theta}{Cos\theta-Sin\theta}$. **(3 times)**

2. Find the values of remaining trigonometric function if $\tan\theta = \frac{-1}{3}$ and the terminal arm of the angle is in quadrant II.

3. Prove that: $(\tan\theta + \cot\theta)^2 = \sec^2\theta\ \text{cosec}^2\theta$.

5. If $\tan\theta = \frac{1}{\sqrt{7}}$ & angle is not in the III quadrant. Find value $\frac{cosec^2\theta - \sec^2\theta}{cosec^2 + sec^2\theta}$ **(2 times)**

6. Prove that $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ When θ is 30°.

7. Prove the following identity $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$ **(2 times)**

8. Prove that $\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1 - \sin^2.\cos^2\theta)$ **(5 times)**

9. Prove that $\frac{1}{cosec\theta-cot\theta} - \frac{1}{sin\theta} = \frac{1}{sin\theta} - \frac{1}{cosec\theta+cot\theta}$ **(2 times)**

10. Show that $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2sec^2\theta$ **(2 times)**

11. Prove that $\frac{tan\theta+sec\theta-1}{tan\theta+sec\theta+1} = \tan\theta + \sec\theta$ **(3 times)**

12. Prove that $\ell = r\theta$

13. Find the value of the trigonometric functions of the angle 675°.

14. Find the values of the trigonometric functions of the angle $\theta = \frac{-17}{3}\pi$

(3 Times)

15. If $\sin\theta = -\frac{1}{\sqrt{2}}$ and the teinal a of angle is not in quad. III Find the values of remaining trigonometric functions.

16. Prove that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$ where θ is not an odd multiple of $\frac{\pi}{2}$

17. Two cities A and B lies on the equator, such that their longitudes are 45° E and 25° W respectively. Find the distance between the two cities, taking the radius of the earth as 6400 kms.

18. Prove that $\frac{1+\cos\theta}{1-\cos\theta} = (cosec\theta + \cot\theta)^2$

19. If $cosec\theta = \frac{m^2+1}{2m}$ and $m > 0\left(0 < \theta < \frac{\pi}{2}\right)$ find the values of the remaining trigonometric ratios.

OBJECTIVE MCQ'S OF CHAPTER-10 IN ALL PUNJAB BOARDS 2011-2021

Topic I: Fundamental Law of Trignometric:

1. Cosec $\left(\pi/2 + \theta\right)$ = ______ (3 times)
(A) Secθ (B)- Secθ (C) Co secθ (D) $-$Co secθ

2. Cos $(\alpha+\beta)$ – Cos $(\alpha-\beta)$ = (2 times)
(A) 2Sin α Cos β (B) Cos α Sin β (C) 2Cos α Cos β (D) -2Sin α Sin β

3. Sin $\left(\alpha - \frac{\pi}{2}\right)$ = (3 times)
(A) sec α (B) $-$ cos α (C) cos α (D) $-$ Sin α

4. If θ is an acute angle then $\frac{3\pi}{2} + \theta$ lies in quadrant:- (3 times)
(A) I (B) II (C) III (D) IV

5. cos $\left(a - \pi/2\right)$ = (1 time)
(A) $-$sin a (B) -cos a (C) cos a (D) sin a

6. Tan $(45° - \theta)$ = (3 times)
(A) $\frac{\tan\theta+1}{\tan\theta-1}$ (B) $\frac{1+\tan\theta}{1-\tan\theta}$ (C) $\frac{1+\tan^2\theta}{1-\tan^2\theta}$ (D) $\frac{1-\tan\theta}{1+\tan\theta}$

7. sin 390° = (4 times)
(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $-\frac{1}{\sqrt{2}}$ (D) 1

8. Sin $(45° + \alpha)$ = (4 times)
(A) sin α + cos α (B) sin α - cos α (C) $\frac{1}{\sqrt{2}}$(sin α + cos α) (D) cos α- sin α

9. Sin (90° + θ) = (2 times)
(A) sin θ (B) cos θ (C) -sin θ (D) - cos θ

10. Cos $\left(\pi/2 - \beta\right)$is equal to:
(A) sin β (B) -sin β (C) cos β (D) -cos β

11. Sin $\left(\frac{3\pi}{2} - \theta\right)$ is equal to: (3 times)
(A) Sinθ (B) -Sinθ (C) Cosθ (D) $-$Cosθ

12. Cot $\left(\frac{3\pi}{2} - \theta\right)$ is equal to:
(A) tanθ (B) $-$tan θ (C) cotθ (D) None of these

13. Sin $(\theta - \frac{\pi}{2})$=
(A) cosθ (B) - cosθ (C) sinθ (D) cosecθ

14. sin $(\theta + 270°)$ = ______ (2 times)
(A) cot θ (B) tan θ (C) sin θ (D) $-$ cos θ

15. Cot $(\pi - \alpha)$ =
(A) Sin α (B) Cot α (C) $-$Cot α (D) tan α

16. The value of cos 315° is:
(A) 0 (B) 1 (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{2}}$

17. Tan $(\pi - \alpha)$ is equal to:
(A) tan α (B) $-$ tanθ (C) $-$ tanα (D) cot α

Topic II: Half, Double and Tripple angle Indenties:

18. Tan2θ = : (2 times)
(A) $\frac{2\tan\theta}{1+\tan^2\theta}$ (B) $\frac{\tan\theta}{1-\tan^2\theta}$ (C) $\frac{2\tan\theta}{1-\tan^2\theta}$ (D) $\frac{1-\tan^2\theta}{1+\tan^2\theta}$

19. 4 cos$^3\alpha$ - 3cos α = (5 times)
(A) Sin 3 α (B) Cos 3 α (C) Cos 2 α (D) Sin 2 α

20. $\sin 2\theta =$ (4 times 2018)
(A) $2\sin\theta.\cos\theta$ (B) $2\tan\theta$ (C) $\sec^2\theta$ (D) $2\sin\theta$

21. $\pm\sqrt{\frac{1-\cos\alpha}{2}} =$ (2 times)
(A) $\cos\frac{\alpha}{2}$ (B) $\tan\frac{\alpha}{2}$ (C) $\sin\alpha$ (D) $\sin\frac{\alpha}{2}$

22. $1+\cos 2\theta$ is equal to:
(A) $2\sin^2\theta$ (B) $2\cos^2\theta$ (C) $2\sin^2\theta/2$ (D) $2\cos^2\theta/2$

23. $\sin 3\alpha$ equals to:
(A) $3\sin\alpha - 4\sin^3\alpha$ (B) $3\sin\alpha + 4\sin^3\alpha$ (C) $4\sin^3\alpha - 3\sin\alpha$ (D) $4\sin^3\alpha + 3\sin\alpha$

24. $\frac{1+\cos\theta}{\sin\theta} =$:
(A) $\sin\theta$ (B) $\cot\frac{\theta}{2}$ (C) $\operatorname{cosec}^2\theta$ (D) $\tan\frac{\theta}{2}$

25. $\frac{1-\tan^2 a}{1+\tan^2 a}$ is equal to: (2 times 2018)
(A) $\cos 2a$ (B) $\sin 2a$ (C) $\cos^2 2a$ (D) $\sin^2 2a$

26. $2\sin^2\theta/2$ equals: (3 times 2018)
(A) $1+\cos\theta$ (B) $1-\cos\theta$ (C) $1+\sin\theta$ (D) $1-\sin\theta$

27. $2\cos^2\theta/2$ equals:
(A) $1+\cos\theta$ (B) $1-\cos\theta$ (C) $1-\sin\theta$ (D) $1+\sin\theta$

28. $\sin\alpha/2$ is equal to:
(A) $\sqrt{\frac{1+\sin\alpha}{2}}$ (B) $\sqrt{\frac{1-\cos\alpha}{2}}$ (C) $\sqrt{\frac{1+\cos\alpha}{2}}$ (D) $\sqrt{\frac{1-\sin\alpha}{2}}$

29. $\sin\theta$ equals:
(A) $2\sin^2\theta/2$ (B) $2\sin\theta/2\cos\theta/2$ (C) $2\cos^2\theta/2$ (D) $2\tan\theta/2$

Topic III: Sum, Difference and Product Sines and Cosines:

30. $2\cos\alpha\sin\beta$ is equal to: (4 times)
(A) $\sin(\alpha+\beta)-\sin(\alpha-\beta)$ (B) $\sin(\alpha+\beta)+\sin(\alpha-\beta)$
(C) $\cos(\alpha+\beta)+\sin(\alpha-\beta)$ (D) $\cos(\alpha+\beta)-\cos(\alpha-\beta)$

31. $\cos(\alpha+\beta)+\cos(\alpha-\beta)$ equal to:- (5 times)
(A) $2\sin\alpha\cos\beta$ (B) $2\cos\alpha\sin\beta$ (C) $2\cos\alpha\cos\beta$ (D) $-2\sin\alpha\sin\beta$

32. $2\cos 5\theta\sin 3\theta =$
(A) $\sin 4\theta - \sin\theta$ (B) $\sin 4\theta + \sin\theta$ (C) $\sin 8\theta - \sin 2\theta$ (D) $\cos 8\theta + \cos 2\theta$

2018

33. The value of $\cos 75° =$
(a) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (b) $\frac{\sqrt{-3}+1}{2\sqrt{2}}$ (c) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (d) $\frac{-\sqrt{3}-1}{2\sqrt{2}}$

34. $\cos(\theta+\frac{3\pi}{2})$ is equal to:
(a) $-\sin\theta$ (b) $\sin\theta$ (c) $-\cos\theta$ (d) $\cos\theta$

35. $\cos(\pi+\theta)$ is equal to:
(a) $\sec\theta$ (b) $-\cos\theta$ (c) $\cos\theta$ (d) $-\sec\theta$

2019

36. $\tan\frac{\alpha}{2} =$:
(a) $\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$ (b) $\pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}$ (c) $\pm\sqrt{\frac{1-\cos\alpha}{2}}$ (d) $\pm\sqrt{\frac{1+\cos\alpha}{2}}$

37. $\sin(-\alpha) =$:
(a) $\sec\alpha$ (b) $-\sin\alpha$ (c) $\sin\alpha$ (d) $-\cos\alpha$

38. **Cos (θ – 180°) = :**

(a) sin θ (b) -Cos θ (c) Cos θ (d) -sin θ

39. **The angle $\frac{3\pi}{2}-\theta$ lies in quadrant:**

(a) I (b) II (c) III (d) IV

40. **If $6\cos^2\theta+2\sin^2\theta=5$, then $\tan^2\theta$ will be equal to:**

(a) $\frac{3}{2}$ (b) 3 (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

41. $\tan\left(\frac{3\pi}{2}+\theta\right)=$

(a) cotθ (b) tanθ (c) -cotθ (d) –tanθ

42. $\cos\left(\frac{3\pi}{2}-\theta\right)=$ **is equal to:**

(a) $-\sin\theta$ (b) $\sin\theta$ (c) $\cos\theta$ (d) $-\cos\theta$

43. $\sin 8\theta-\sin 4\theta=$

(a) $2\sin 6\theta\sin 4\theta$ (b) $2\cos 2\theta\sin 6\theta$ (c) $2\cos 6\theta\sin 2\theta$ (d) $-2\sin 6\theta\cos 2\theta$

2021

44. $\sin\left(180^{\circ}+\alpha\right)=$

(a) $-\cos\alpha$ (b) $\sin\alpha$ (c) $\cos\alpha$ (d) $-\sin\alpha$

45. If $\sin\alpha=4/5, 0<\alpha<\pi/2 \, then \cos\alpha=$

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{4}{5}$ (d) $\frac{3}{5}$

46. **Sin (-300°)**

(a) $\frac{-\sqrt{3}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{2}}$

47. $\sin\left(\frac{\pi}{2}+\alpha\right)$ **equals to**

(a) $-\cos\alpha$ (b) $\sin\alpha$ (c) $\cos\alpha$ (d) $-\sin\alpha$

48. **If $\alpha=30^{\circ}$, then value of $\cot 3\alpha$ =**

(a) 0 (b) 1 (c) 3 (d) ∞

49. $\cos\left(-60^{\circ}\right)=$

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{-\sqrt{3}}{2}$

50. $\cos 2\alpha=$

(a) $2\sin^2\alpha-1$ (b) $2\cos^2\alpha-1$ (c) $2\cos\frac{\alpha}{2}\sin\frac{\alpha}{2}$ (d) $1-2\cos^2\alpha$

51. If $\sin\alpha=\frac{2}{3}, \cos\alpha=\frac{3}{4}, then\ value\ of\ \sin 2\alpha=$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{15}{144}$ (d) 1

ANSWERS TO THE MULTIPLE CHOICE QUESTIONS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
A	D	B	D	D	D	A	C	B	A	D	A	B	D	C	D	C	C
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
B	A	D	B	A	B	A	B	A	B	B	A	C	C	A	B	B	A
37	38	39	40	41	42	43	44	45	46	47	48	49	50	51			
B	B	C	C	C	A	C	D	D	C	B	A	A	B	D			

SHORT QUESTION'S OF CHAPTER-10 IN ALL PUNJAB BOARDS 2011-2021

Topic I: Fundamental Law of Trignometric:

1. Find the value of Sin(-330°) and Cos(-330°) without using table/calculator. (2 times)

Sol. Sin (-330°)

= -sin 330° $\because \sin(-\theta) = -\sin\theta$

= - sin (360° – 30°) = - sin (2π - 30°)

= -[-sin 30°] =sin30° $\because \sin 30° = \frac{1}{2}$

$= +\frac{1}{2}$

Now cos (-330°) $\because \cos(-\theta) = \cos\theta$

= cos 330°= cos (360° – 30°)=

= cos 30° $\because \cos 30° = \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2}$

2. Show that $\frac{Cos11° + Sin11°}{Cos11° - Sin11°}$ = Tan56°. (2 times)

Sol. R.H.S = tan 56°

= tan (45° + 11°)

$\because \tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \frac{\tan 45° + \tan 11°}{1 - \tan 45° \tan 11°} = \frac{1 + Sin11°/Cos11°}{1 - Sin11°/Cos11°}$

$= \frac{\frac{Cos11° + Sin11°}{Cos11°}}{\frac{Cos11° - Sin11°}{Cos11°}} = \frac{Cos11° + Sin11°}{Cos11° + Sin11°}$

= L.H.S (Hence Proved)

3. State the Fundamental law of Trigonometry.

Sol. Fundamental law of trigonometry:

Let α and β any two angles (real numbers) then

Cos ($\alpha - \beta$) = Cos α Cos β + Sin α Sin β

Which is called the fundamental law of trigonometry.

4. Prove that: Sin (180° + α) Sin(90°- α) = - Sin αCosα. (3 times)

Sol. L.H.S = Sin (180° + ∝) Sin (90° - ∝)

We know that in Allied angle

= Sin (180° + ∝) = - *Sin* ∝

∵ Sin (90° - ∝) = Cos ∝

= (- Sin ∝) (Cos ∝)= - Sin ∝ Cos ∝= R.H.S

so . L.H.S = R.H.S

5. Express in the form of r Sin(θ + Ø) : if 12 Sinθ + 5cosθ

Sol. 12 Sin θ + 5 Cosθ

Let 12 = r Cos Ø, 5 = rsinØ

Then by squaring and adding

$(12)^2 + (5)^2 = r^2 Cos^2Ø + r^2 Sin^2Ø$

$144 + 25 = r^2 (Cos^2Ø + Sin^2Ø)$

$169 = r^2$

$$\frac{\sin\phi}{\cos\phi}=\frac{5}{12}$$

$$\tan\phi=\frac{5}{12}$$

$\Rightarrow$ r = 13 and $\tan\emptyset = \frac{r\sin\phi}{r\cos\phi}=\frac{5}{12} \Rightarrow \phi=\tan^{-1}\left(\frac{5}{12}\right)$

12 Sin∅ + 5 Cos∅

= r Cos ∅ Sinθ + r Sin∅ Cosθ

= r[Sinθ Cos∅ + Cosθ Sin∅]

= r Sin (θ + ∅) $\quad \therefore$ Sin ($\alpha + \beta$) = Sin α Cos β + Cosα Sinβ

Where r= 13 & ∅ = $\tan^{-1}$ 5/12

6. **Prove that $\frac{Sin(\alpha+\beta)+Sin(\alpha-\beta)}{Cos(\alpha+\beta)+Cos(\alpha-\beta)}$ = tanα**

Sol. $\frac{Sin(\alpha+\beta)+Sin(\alpha-\beta)}{Cos(\alpha+\beta)+Cos(\alpha-\beta)}$ = tanα

L.H.S = $\frac{Sin(\alpha+\beta)+Sin(\alpha-\beta)}{Cos(\alpha+\beta)+Cos(\alpha-\beta)}$

$=\frac{Sin\,\alpha\,Cos\beta+Cos\,\alpha\,Sin\,\beta+Sin\,\alpha\,Cos\,\beta-Cos\,\alpha\,Sin\,\beta}{Cos\,\alpha\,Cos\,\beta-Sin\,\alpha\,Sin\beta+Cos\,\alpha\,Cos\,\beta+Sin\,\alpha\,Sin\,\beta}$

$=\frac{2Sin\,\alpha\,Cos\,\beta}{2\,Cos\,\alpha\,Cos\,\beta}=\frac{Sin\,\alpha}{Cos\,\alpha}$ = tan α = R.H.S(Hence Proved)

7. **Prove that Cos (α + 45°) = $\frac{1}{\sqrt{2}}$(Cosα − Sinα).** **(2 times 2018)**

Sol. Cos (α + 45°) = $\frac{1}{\sqrt{2}}$(Cosα − Sinα)

L.H.S = Cos (α + 45°) = Cosα Cos45° − Sinα Sin45°

= Cosα $\frac{1}{\sqrt{2}}$ − Sinα $\frac{1}{\sqrt{2}}$ = $\frac{1}{\sqrt{2}}$(Cosα − Sinα)

= R.H.S (Hence Proved)

8. **If α , β , γ are the angles of a triangle ABC , then prove that Cos $\left(\frac{\alpha+\beta}{2}\right)$ = sin$\frac{\gamma}{2}$.** **(4 times)**

Sol. Cos $\left(\frac{\alpha+\beta}{2}\right)$ = sin$\frac{\gamma}{2}$

$\because \alpha+\beta+\gamma=180°$

L.H.S = Cos $\left(\frac{\alpha+\beta}{2}\right)$

$\because \alpha+\beta=180°-\gamma$

$\therefore \frac{\alpha+\beta}{2}=\frac{180°-\gamma}{2}$

= Cos $\left(\frac{180°-\gamma}{2}\right)$

= Cos $\left(\frac{180°}{2}-\frac{\gamma}{2}\right)$ $\Rightarrow$ = Cos $\left(90°-\frac{\gamma}{2}\right)$

= Sin $^{\gamma}/_{2}$= R.H.S

($\because$ Cos (90° − θ) = Sin θ

Hence Proved.

9. **Prove that sin ($\alpha - \beta$) sin ($\alpha + \beta$) = sin² α − sin² β.** **(3 times)**

Sol. L.H.S = sin ($\alpha + \beta$) sin ($\alpha - \beta$)

= (sin α cos β + cos α sin β) (sin α cos β - cos α sin β)

= $\sin^2\alpha\cos^2\beta$ + $Cos^2\alpha\sin^2\beta$

= $\sin^2\alpha\,(1-Sin^2\beta)-(1-\sin^2\alpha)\sin^2\beta$

$= \sin^2\alpha - \sin^2\alpha\sin^2\beta - \sin^2\beta + \sin^2\alpha\sin^2\beta = \sin^2\alpha - \sin^2\beta$

= R.H.S Proved

10. Without using table/calculator find the value of Sin75° and Cos 75°.

Sol. (a) $\sin 75° = \sin(45° + 30°) = \sin 45° \cos 30° + \cos 45° \sin 30°$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}+1}{2\sqrt{2}}$

(b) $\cos 75° = \cos(45° + 30°) = \cos 45° \cos 30° - \sin 45° \sin 30°$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}-1}{2\sqrt{2}}$

11. If α, β, γ are the angles of triangle ABC, then prove that $\sin(\alpha+\beta) = \sin\gamma$ **(6 times)**

Sol. Since α,β,γ are the angles of a triangle ABC

$\therefore \alpha+\beta+\gamma = 180° \Rightarrow \alpha+\beta = 180° - \gamma$

L.H.S $= \sin(\alpha+\beta)$

$= \sin(180° - \gamma) = \sin(2(90°) - \gamma) = \sin\gamma$ = R.H.S (Hence Proved)

12. Find the values of sin105°. **(2 times)**

Sol. $\sin 105° = \sin(60° + 45°)$

$= \sin 60° \cos 45° + \cos 60° \sin 45°$

$= \frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}} + \frac{1}{2}.\frac{1}{\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

13. Without using table find the value of cot (-855°). **(2 times)**

Sol. $\cot(-855°) = -\cot(855°) = \frac{-1}{\tan(855°)}$

$= \frac{-1}{\tan(810°+45°)} = \frac{-1}{\tan(9(90°)+45°)} = \frac{-1}{-\tan 45°} = \frac{-1}{-1} = 1$

14. Without using tables find the value of sec (-960°).

Sol. $\sec(-960°)$

$= \frac{1}{\cos(-960°)} = \frac{1}{\cos 960°} = \frac{1}{\cos(960°+60°)} = \frac{1}{\cos[10(90°)+60°]}$

$= \frac{1}{-\cos 60°} = \frac{1}{-1/2} = -2$

($\because \theta$ consists of even multiple of 90° and lies in III quadrant)

15. Prove that :- Sin (θ + 270°) = - Cos θ

Sol. $\sin(\theta + 270°) = -\cos\theta$

L.H.S $= \sin(\theta + 270°)$ ($\because \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$)

$= \sin\theta\cos 270° + \cos\theta\sin 270°$

$= \sin\theta(0) + \cos\theta(-1) = 0 - \cos\theta$

$= -\cos\theta$ = R.H.S

(Hence Proved)

16. Prove that Cos 330° Sin600° + Cos120° Sin150° = -1 **(3 times)**

Sol. Take $\cos 330° = \cos(270° + 60°)$

$= \cos(3 \times 90° + 60°)$

$\cos 330° = \sin 60°$

$\cos 330° = \frac{\sqrt{3}}{2}$

$\sin(90° \times 6 + 60°) = -\sin 60° = \frac{-\sqrt{3}}{2}$

$\because \sin 600° = -\frac{\sqrt{3}}{2}$

$\cos 120° = \cos(90° + 30°)$

$= \cos(1 \times 90° + 30°)$

$= -\text{Sin}30^\circ$

$\text{Cos}120^\circ = -\frac{1}{2}$

$\therefore$ L.H.S = Cos330°. Sin600° + Cos120° Sin150°

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{-\sqrt{3}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) = -\frac{3}{4} - \frac{1}{4} = \frac{-3-1}{4} = \frac{-4}{4} = -1$

17. Prove that Tan (45° + A). Tan (45° – A) = 1 **(6 times 2021)**

Sol. L.H.S = Tan (45° + A). Tan (45° – A)

$= \text{Tan}(\alpha + \beta) . \text{Tan}(\alpha - \beta)$

$= \left(\frac{Tan45^o + TanA}{1 - Tan45^oTanA}\right)\left(\frac{Tan45^o - TanA}{1 + Tan45^oTanA}\right)$

$= \left(\frac{1+Tan\ A}{1-1\ Tan\ A}\right)\left(\frac{1-Tan\ A}{1+1.Tan\ A}\right) = \left(\frac{1+Tan\ A}{1-1\ Tan\ A}\right)\left(\frac{1-Tan\ A}{1+1.Tan\ A}\right) = 1 =$ R.H.S

18. Prove that Cos306° + Cos234° + Cos162° + Cos18° = 0

Sol. Cos360° = Cos (270° + 36°)

Cos306° = Cos (3 x 90° + 36°)

Cos306° = Sin36°

Cos 234° = Cos (270° – 36°)

Cos 234° = Cos(3 x 90° – 36°)

Cos234° = - Sin36°

Cos162° = Cos (180° – 18°)

= Cos (2 x 90° – 18°)

Cos 162° = - Cos18°

$\therefore$ L.H.S = Cos306° + Cos234° + Cos162° + Cos 18°

= Sin36° – Sin36° – Cos18° + Cos18°

= 0= R.H.S

19. Prove that Sin780° Sin480° + Cos120° Sin30° = $\frac{1}{2}$

Sol. Sin780° = Sin (720° + 60°)

= Sin (2 x 360° + 60°)

Sin 780° = Sin60°

$\text{Sin}780^\circ = \frac{\sqrt{3}}{2}$

Sin 480° = Sin (360° + 120°)

= Sin (1 x 360° + 120°)

= Sin 120°

= Sin (90° + 30°)

Sin 480° = Sin (1 x 90° + 30°)

Sin 480° = Cos30°

$\text{Sin } 480^\circ = \frac{\sqrt{3}}{2}$

Cos (120°) = Cos (90° + 30°)

= Cos (1 x 90° + 30°)

Cos 120° = -Sin30°

$\text{Cos}120^\circ = -\frac{1}{2}$

$\therefore$ L.H.S

= Sin780°Sin480°+Cos120°.Sin30°

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{(\sqrt{3})^2}{4} - \frac{1}{4} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2} =$ R.H.S

20. Prove that Sin $(\theta + \frac{\pi}{2})$ = Cos θ **(4 times 2018)**

Sol. We know

$\text{Sin}(\propto + \beta) = \text{Sin}\propto Cos\beta + Cos \propto Sin\beta$

$\text{Sin}\left(\theta + \frac{\bar{\wedge}}{2}\right) = \text{Sin}\theta\ \text{Cos}\frac{\bar{\wedge}}{2} + \text{Cos Sin}\frac{\bar{\wedge}}{2}$

$\text{Sin}\left(\theta + \frac{\bar{\wedge}}{2}\right) = \text{Sin}\theta\ (0) + \text{Cos}\theta\ (1)$

$\text{Sin}\left(\theta + \frac{\bar{\wedge}}{2}\right) = \text{Cos}\theta$

21. **Prove that** $\frac{1-Tan\theta\, Tan\,\varphi}{1+Tan\theta\, Tan\,\varphi} = \frac{Cos\,(\theta+\varphi)}{Cos\,(\theta-\varphi)}$ **(2 times)**

Sol. L.H.S = $\frac{1-Tan\theta\, Tan\,\varphi}{1+Tan\theta\, Tan\,\varphi}$

$= \frac{1-\frac{Sin\theta}{Cos\theta}\frac{Sin\varphi}{Cos\,\varphi}}{1+\frac{Sin\theta}{Cos\theta}\frac{Sin\varphi}{Cos\,\varphi}} = \frac{\frac{Cos\theta Cos\varphi - Sin\theta Sin\varphi}{Cos\theta Cos\varphi}}{\frac{Cos\theta Cos\varphi + Sin\theta Sin\varphi}{Cos\theta Cos\varphi}}$

$= \frac{Cos\theta Cos\varphi - Sin\theta Sin\varphi}{Cos\theta Cos\varphi + Sin\theta Sin\varphi} = \frac{Cos\,(\theta+\varphi)}{Cos\,(\theta-\varphi)} =$ R.H.S

22. **Prove that: Sin $(\propto + \beta)$ - Sin$(\propto - \beta)$ = 2Cos$\propto$ Sinβ**

Sol. L.H.S = Sin$(\propto + \beta)$ - Sin $(\propto - \beta)$

= (Sin $\propto$ Cosβ + Cos $\propto$ Sinβ) – (Sin $\propto$ Cosβ – Cos $\propto$ Sinβ)

= Sin$\propto$Cosβ + Cos$\propto$Sinβ - Sin$\propto$Cosβ + Cos$\propto$Sinβ= 2Cos$\propto$ Sinβ= R.H.S

23. **Find the distance between two points P(Cosx , Cosy) and Q (Sinx , Siny)**

Sol. Let $P(x_1, y_1)$ = P(Cosx , Cosy)

$Q(x_2, y_2)$ = Q (Sinx , Siny)

Then By distance formula

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)_2} = \sqrt{(Sinx - Cosx)^2 + (Siny - Cosy)^2}$

$d = \sqrt{Sin^2x + Cos^2x - 2SinxCosx + Sin^2y + Cos^2y - 2Siny\, Cosy}$

$d = \sqrt{1 - 2SinxCosx + 1 - 2\,Siny\, Cosy} = \sqrt{2 - 2\,Sinx\,Cosx - 2Siny\,Cosy}$

$d = \sqrt{2 - 2(SinxCosx + SinyCosy}$

24. **If $\propto$, β and r are angles of Triangle ABC, Prove that Tan $(\propto + \beta)$ + Tan r = 0** **(3 times)**

Sol. $\propto + \beta + r = 180°$

$\propto + \beta = 180° - \gamma$

tan $(\propto + \beta)$ = Tan (180° –γ)

tan $(\propto + \beta)$ = Tan (2 x 90° –γ)

tan $(\propto + \beta)$ = -Tan γ

tan $(\propto + \beta)$ + Tan γ = 0

25. **Prove that: Cos $(\propto + 45^o) = \frac{1}{\sqrt{2}}\ (Cos \propto\ - Sin \propto)$ (7 times 2018)**

Sol. We know Cos $(\propto + \beta)$ = Cos$\propto$ $Cos\beta - Sin \propto Sin\beta$

L.H.S = Cos $(\propto + 45^o)$ = Cos$\propto$ $Cos45^o - Sin \propto Sin45^o$

= Cos$\propto \frac{1}{\sqrt{2}} - Sin \propto \frac{1}{\sqrt{2}}$

Cos $(\propto + 45^o) = \frac{1}{\sqrt{2}}$ (Cos$\propto$ $- Sin \propto$) = R.H.S

26. **Prove that** $\frac{Cos\,8^o - Sin8^o}{Cos8^o + Sin8^o}$ **= Tan37°** **(2 times)**

Sol. R.H.S = Tan37°

R.H.S = Tan (45° – 8°)

$= \frac{Tan45^o - Tan8^o}{1 + Tan45^o\, Tan8^o}$

$= \frac{1-\frac{Sin\,8^o}{Cos\,8^o}}{1+1\frac{Sin8^o}{Cos8^o}} = \frac{\frac{Cos8^o - Sin8^o}{Cos8^o}}{\frac{Cos8^o + Sin8^o}{Cos8^o}} = \frac{Cos8^o - Sin8^o}{Cos8^o + Sin8^o} =$ L.H.S

27. **Show that**

$\cos(\alpha + \beta)\cos(\alpha - \beta) = \cos^2\alpha - \sin^2\beta$ **(2 times)**

Sol $L.H.S = \cos(\alpha + \beta) . \cos(\alpha - \beta)$

$= (\cos\alpha \cos\beta - \sin\alpha \sin\beta)(\cos\alpha \cos\beta + \sin\alpha \sin\beta)$

$= (\cos\alpha \cos\beta)^2 - (\sin\alpha \sin\beta)^2 = \cos^2\alpha \cos^2\beta - \sin^2\alpha \sin^2\beta$

$= \cos^2\alpha \cos^2\beta - \sin^2\alpha \sin^2\beta = \cos^2\alpha(1 - \sin^2\beta) - (1 - \cos^2\alpha)\sin^2\beta$

$= \cos^2\alpha - \cos^2\alpha \sin^2\beta - \sin^2\beta + \cos^2\alpha \sin^2\beta = \cos^2\alpha - \sin^2\beta = R.H.S$

28. **Prove that** $\tan(180^0 + \theta) = \tan\theta$ **(3 times)**

Sol Given $\tan(180^0 + \theta) = \tan\theta$

L.H.S $= \tan(180^0 + \theta)$

$\because \tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$

$= \dfrac{\tan 180^0 + \tan\theta}{1 - \text{tna}180^0 \tan\theta}$ $\because \tan 180^0 = 0$

$= \dfrac{0 + \tan\theta}{1 - 0} = \tan\theta = R.H.S$

L.H.S = R.H.S

29. **Show that** $\cos\left(\dfrac{\pi}{2} - \beta\right) = \sin\beta$

Sol $\cos\left(\dfrac{\pi}{2} - \beta\right) = \sin\beta$

$L.H.S = \cos\left(\dfrac{\pi}{2} - \beta\right)$ $\because \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \cos\dfrac{\pi}{2}\cos\beta + \sin\dfrac{\pi}{2}\sin\beta$ $\because \cos\dfrac{\pi}{2} = 0$

$= 0\cos\beta + 1\sin\beta$ $\because \sin\dfrac{\pi}{2} = 1$

$= 0 + \sin\beta$

$= \sin\beta$

= R.H.S

L.H.S = R.H.S

30. **Find the value of sec(-300⁰)(without table).**

Sol $\sec(-300)$

$= \sec(300)$ $\because \cos 60^0 = \dfrac{1}{2}$

$= \sec(360 - 60)$

$= \sec 60$ $\because \sec 60^0 = 2$

$= 2$

31. **Prove that** $\cos(360^0 - \theta) = \cos\theta$

Sol $\cos(360^0 - \theta) = \cos\theta$

$L.H.S = \cos(360^0 - \theta)$ $\because \cos(\alpha - \beta)\cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \cos 360^0 \cos\theta + \sin 360^0 \sin\theta$

$= 1.Cos\theta + 0.Sin\theta$

$= Cos\theta$

$=$ R.H.S

L.H.S = R.H.S

Topic II: Half, Double and Tripple angle Indenties:

32. Prove that : $1 + \tan\alpha \tan 2\alpha = \text{Sec } 2\alpha$. **(4 times)**

Sol. L.H.S = $1 + \tan\alpha \tan 2\alpha$

$$= 1 + \frac{Sin\,\alpha}{Cos\,\alpha} \cdot \frac{Sin\,2\alpha}{Cos\,2\alpha} = \frac{Cos\,\alpha\ Cos\,2\alpha + Sin\,\alpha\ Sin\,2\alpha}{Cos\,\alpha\ Cos\,2\alpha}$$

$\because Cos\,(\alpha - \beta) = Cos\,\alpha\ Cos\,\beta + Sin\,\alpha\ Sin\,\beta$

$$\text{L.H.S} = \frac{Cos\,(2\alpha - \alpha)}{Cos\,\alpha\ Cos\,2\alpha} \Longrightarrow = \frac{Cos\,\alpha}{Cos\,\alpha\ Cos\,2\alpha} = \frac{1}{Cos\,2\alpha} = \sec 2\alpha = \text{R.H.S}$$

34. Write formulas for $\cos 2\alpha$ and $\tan 2\alpha$ in terms of α.

Sol. Formulas for

- $Cos2\alpha = Cos^2\alpha - Sin^2\alpha \quad = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$
- $\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$

35. Prove that $\frac{\sin 3x - \sin x}{\cos x - \cos 3x} = \cot 2x$

Sol. $\frac{\sin 3x - \sin x}{\cos x - \cos 3x} = \cot 2x$

$$\text{L.H.S} = \frac{\sin 3x - \sin x}{\cos x - \cos 3x}$$

$$= \frac{2\,Cos\left(\frac{3x+x}{2}\right) Sin\left(\frac{3x-3}{2}\right)}{-2\,Sin\left(\frac{3x+x}{2}\right) Sin\left(\frac{x-3x}{2}\right)} = \frac{2\,Cos\,2x\ Sin\,x}{-2\,Sin\,2x\ Sin\,(-x)}$$

$$= \frac{Cos\,2x\ Sin\,x}{Sin\,2x\ Sin\,x} \qquad (\therefore Sin\,(-x) = -Sin\,x)$$

$$= \frac{Cos\,2x}{Sin\,2x} = Cot\,2x$$

= R.H.S (Hence Proved)

36. Show that $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$ **(2 times 2018)**

Sol. $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

L.H.S = $\sin 2\theta = 2\sin\theta\cos\theta$

$$= \frac{2\sin\theta\cos\theta}{1} = \frac{2\sin\theta\cos\theta}{\cos^2\theta + \sin^2\theta}$$

$$= \frac{\frac{2\sin\theta\cos\theta}{\cos^2\theta}}{\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}} = \frac{2\frac{\sin\theta}{\cos\theta}}{1 + \frac{\sin^2\theta}{\cos^2\theta}}$$

$$\text{Sin}2\,\theta = \frac{2\tan\theta}{1 + \tan^2\theta}$$

= R.H.S Proved.

37. Prove that $\sin 3\alpha = 3\,Sin\,\alpha - 4\sin^3\alpha$ **(2 times 2018)**

Sol. $Sin3\alpha = 3\,Sin\,\alpha - 4\sin^3\alpha$

L.H.S = $\sin 3\alpha = \sin(2\alpha + \alpha)$

$= \sin 2\alpha \cos\alpha + \cos 2\alpha \sin\alpha$

$= 2\sin\alpha\cos\alpha\cos\alpha + (1 - 2\sin^2\alpha)\sin\alpha$

$= 2\sin\alpha\cos^2\alpha + \sin\alpha - 2\sin^3\alpha$

$= 2\sin\alpha(1 - \sin^2\alpha) + \sin\alpha - 2\sin^3\alpha$

$= 2\sin\alpha - 2\sin^3\alpha + \sin\alpha - 2\sin^3\alpha$

$= \sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha = R.H.S$ (Hence Proved)

38. **Find the value of $\sin 2\alpha$ for $\sin\alpha = 12/13$; where $0 < \alpha < \pi/2$** **(2 times)**

Sol. $\sin\alpha = 12/13$; where $0 < \alpha < \pi/2$

$\Rightarrow$ α is in 1st quad.

$\because \cos^2\alpha + \sin^2\alpha = 1$

$\therefore \cos^2\alpha = 1 - \sin^2\alpha$

$= 1 - \frac{144}{169} = \frac{169-144}{169} = \frac{25}{169}$

$\cos\alpha = \pm\frac{5}{13} = \frac{5}{13}$

Now $\sin 2\alpha = 2.\left(\frac{12}{13}\right)\left(\frac{5}{13}\right)$

$\sin 2\alpha = \frac{120}{169}$ $\quad \because \alpha \in 1st\ Quad$

39. **Prove that $\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2.$** **(4 times)**

Sol. $L.H.S = \frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = \frac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\sin\theta\cos\theta}$

$= \frac{\sin(3\theta-\theta)}{\sin\theta\cos\theta} = \frac{\sin 2\theta}{\sin\theta\cos\theta} = \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2 = R.H.S$

40. **When $\cos\alpha = 3/5$ $0 < \alpha < \frac{\pi}{2}$ Find value of $\cos 2\alpha$**

Sol. $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$\cos 2\alpha = \cos^2\alpha - (1 - \cos^2\alpha)$

$\cos 2\alpha = \cos^2\alpha - 1 + \cos^2\alpha$

$\cos 2\alpha = 2\cos^2\alpha - 1$

$\cos 2\alpha = 2\left(\frac{3}{5}\right)^2 - 1 = 2\left(\frac{9}{25}\right) - 1 = \frac{18}{25} - 1$

$\cos 2\alpha = \frac{18-25}{25} = -\frac{7}{25}$

41. **Prove that $\frac{\sin 3x - \sin x}{\cos x - \cos 3x} = \cot 2x$**

Sol. $L.H.S = \frac{\sin 3x - \sin x}{\cos x - \cos 3x}$

$= \frac{\sin 3x - \sin x}{-(-\cos x + \cos 3x)} = -\left[\frac{\sin 3x - \sin x}{\cos 3x - \cos}\right]$

$L.H.S = -\left[\frac{\sin\alpha - \sin\beta}{\cos x - \cos\beta}\right]$

$= -\left[\frac{2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}{-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}\right] = \frac{\cos\left(\frac{\alpha+\beta}{2}\right)}{\sin\left(\frac{\alpha+\beta}{2}\right)} = \frac{\cos\left(\frac{3x+x}{2}\right)}{\sin\left(\frac{3x+x}{2}\right)} = \frac{\cos\left(\frac{4x}{2}\right)}{\sin\left(\frac{4x}{2}\right)}$

$= \frac{\cos 2x}{\sin 2x} = \cot 2x = R.H.S$

42. **Prove that: $\sin\left(\frac{\pi}{4} - \theta\right).\sin\left(\frac{\pi}{4} + \theta\right) = \frac{1}{2}\cos 2\theta$** **(3 times)**

Sol. $L.H.S = \sin\left(\frac{\pi}{4} - \theta\right)\sin\left(\frac{\pi}{4} + \theta\right)$

$= (\sin\frac{\pi}{4}\cos\theta - \cos\frac{\pi}{4}\cos\theta)(\sin\frac{\pi}{4}\cos\theta - \cos\frac{\pi}{4}\cos\theta)$

$= (\frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta)(\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta) = \frac{1}{\sqrt{2}}(\cos\theta - \sin\theta)\frac{1}{\sqrt{2}}(\cos\theta + \sin\theta)$

$= \left(\frac{1}{\sqrt{2}}\right)^2(\cos\theta - \sin\theta)(\cos\theta + \sin\theta) = \frac{1}{2}[\cos^2\theta - \sin^2\theta] = \frac{1}{2}\cos 2\theta = R.H.S$

43. **Prove that** $\text{Cos}2\theta = \frac{1-Tan^2\theta}{1+Tan^2\theta}$

Sol. R.H.S $= \frac{1-Tan^2\theta}{1+Tan^2\theta}$

$= \frac{1-Tan^2\theta}{Sec^2\theta} = \frac{1}{Sec^2\theta}(1-\text{Tan}^2\theta) = \text{Cos}^2\theta\,(1 - \frac{Sin^2\theta}{Cos^2\theta})$

$= \text{Cos}^2\theta - \text{Sin}^2\theta = \text{Cos}2\theta =$ L.H.S

44. **Prove that:** $\frac{Sin8x+Sin2x}{Cos8x+Cos2x} = Tan5x$ **(4 times)**

Sol. L.H.S $= \frac{Sin8x+Sin2x}{Cos8x+Cos2x}$

$= \frac{Sin\alpha+Sin\beta}{Cos\alpha+Cos\beta}$

$= \frac{2\sin\left(\frac{\alpha+\beta}{2}\right)Cos\left(\frac{\alpha-\beta}{2}\right)}{2\,Cos\left(\frac{\alpha+\beta}{2}\right)Cos\left(\frac{\alpha-\beta}{2}\right)} = \frac{2\,Sin\left(\frac{8x+2x}{2}\right)}{2\,Cos\left(\frac{8x+2x}{2}\right)}$

$= \frac{Sin\,5x}{Cos\,5x} = \tan 5x =$ R.H.S

45. **Prove that** $\frac{Co\sec\theta + 2Co\sec 2\theta}{\sec\theta} = Cot\frac{\theta}{2}$

Sol $\frac{Co\sec\theta + 2Co\sec 2\theta}{\sec\theta} = Cot\frac{\theta}{2}$

$= \frac{Co\sec\theta + 2Co\sec 2\theta}{\sec\theta}$

$= \frac{\frac{1}{Sin\theta} + \frac{2}{Sin2\theta}}{\frac{1}{Cos\theta}}$

$= \left(\frac{1}{Sin\theta} + \frac{2}{Sin2\theta}\right)Cos\theta$

$\because 2\cos^2\theta/_2 = 1+\cos\theta$

$\because Sin\theta = 2Sin\theta/_2\,Cos\theta/_2$

$= \left(\frac{1}{Sin\theta} + \frac{2}{2Sin\theta Cos\theta}\right)Cos\theta$

$= \frac{Cos\theta}{Sin\theta} + \frac{Cos\theta}{Sin\theta Cos\theta}$

$= \frac{Cos\theta}{Sin\theta} + \frac{1}{Sin\theta} = \frac{Cos\theta+1}{Sin\theta} = \frac{1+Cos\theta}{Sin\theta}$

$= \frac{2Cos^2\theta/_2}{2Sin\theta/_2\,Cos\theta/_2}$

$= \frac{Cos\theta/_2}{Sin\theta/_2} = Cot\theta/_2$

= R.H.S

L.H.S = R.H.S

46. Prove that: $Cot(\alpha+\beta)=\dfrac{Cot\alpha Cot\beta-1}{Cot\alpha+Cot\beta}$ **(2 times)**

Sol $Cot(\alpha+\beta)=\dfrac{Cot\alpha Cot\beta-1}{Cot\alpha+Cot\beta}$

$$R.H.S=\frac{Cot\alpha Cot\beta-1}{Cot\alpha+Cot\beta}=\frac{\frac{Cos\alpha Cos\beta}{Sin\alpha Sin\beta}-1}{\frac{Cos\alpha}{Sin\alpha}+\frac{Cos\beta}{Sin\beta}}$$

$$=\frac{\frac{Cos\alpha Cos\beta-Sin\alpha Sin\beta}{Sin\alpha Sin\beta}}{\frac{Cos\alpha Sin\beta+Sin\alpha Cos\beta}{Sin\alpha Sin\beta}}=\frac{Cos\alpha Cos\beta-Sin\alpha Sin\beta}{Sin\alpha Cos\beta+Cos\alpha Sin\beta}$$

$$=\frac{Cos(\alpha+\beta)}{Sin(\alpha+\beta)}$$

$$=Cot(\alpha+\beta)$$

= L.H.S

L.H.S = R.H.S

Topic III: Sum, Difference and Product Sines and Cosines:

47. Prove that Cos20° + Cos100° + Cos140° = 0 ; without using table/calculator. **(2 times)**

Sol. L.H.S = Cos20° + Cos100° + Cos140°

$= 2\ Cos\ \frac{20+100}{2}\ Cos\ \frac{20-100}{2} + Cos140°$

$= 2\ Cos\ \frac{120}{2}\ Cos\ (-\frac{80}{2}) + Cos140°$

$= 2\ Cos60°\ Cos\ (-40) + Cos140°$

$\because Cos\ 60 = \frac{1}{2}$

$\because Cos\ (-40) = Cos\ 40$

$\because Cos140 = Cos\ (180-40) = -Cos\ 40$

$= 2\frac{1}{2}\ Cos\ 40° - Cos\ 40° = Cos\ 40° - Cos40° = 0$

48. Express the following sum as product : Sin 5θ + Sin3 θ. (2 times)

Sol. Sin 5 θ + Sin 3 θ

$\because SinP + Sin\ Q = 2Sin\left(\frac{P+Q}{2}\right) Cos\left(\frac{P-Q}{2}\right)$

$= 2Sin\left(\frac{5\theta+3\theta}{2}\right) Cos\left(\frac{5\theta-3\theta}{2}\right)$

$= 2Sin\left(\frac{8\theta}{2}\right) Cos\left(\frac{2\theta}{2}\right) = 2Sin4\theta\ Cos\theta$

49. Express 2Sin3θ Cos θ as sum or difference. **(2 times)**

Sol. $2\ Sin3\theta\ Cos\theta$

$= Sin\ (3\theta+\theta) + Sin\ (3\theta-\theta)$

$[\because 2\ Sin\alpha\ Cos\beta = Sin\ (\alpha+\beta) + Sin\ (\alpha-\beta)]$

$= Sin4\theta + Sin\ 2\theta$ Ans.

50. Express sin (x + 30°) + sin (x – 30°) as product.

Sol. sin (x + 30°) + sin (x – 30°)

$= 2\ Sin\left(\frac{x+30^o+x-30^o}{2}\right) Cos\left(\frac{x+30^o-x+30^o}{2}\right)$

$= 2\ Sin\left(\frac{2x}{2}\right) Cos\left(\frac{60^o}{2}\right) = 2\ Sinx\ Cos30^o$

51. Express Cos7θ - Cos θ as product. **(2 times)**

Sol. $\cos 7\theta - \cos\theta = -2\sin\left(\frac{7\theta+\theta}{2}\right)\sin\left(\frac{7\theta-\theta}{2}\right)$

$\left(\because \cos P - \cos Q = -2\sin\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)\right)$

$= -2\sin 4\theta \sin 3\theta$

52. Express the product as sum or difference 2cos5θ sin3θ. **(2 times)**

Sol. $2\cos 5\theta \sin 3\theta$

$\{\because 2\cos\alpha \sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)\}$

$\Rightarrow$ $2\cos 5\theta \sin 3\theta = \sin(5\theta + 3\theta) - \sin(5\theta - 3\theta)$

$= \sin 8\theta - \sin 2\theta$

53. Express product as sums or difference cos(2x + 30°) . cos (2x – 30°).

Sol. $\cos(2x + 30^\circ) \cdot \cos(2x - 30^\circ)$

$= \frac{1}{2}[2\cos(2x + 30^o)\cos(2x - 30^o)]$

$= \frac{1}{2}[\cos(2x + 30^o + 2x - 30^o) + \cos(2x + 30^o - 2x + 30^o)]$

$= \frac{1}{2}[\cos 4x - \cos 60^\circ]$

54. Express cos6θ + cos3θ as product.

Sol $Cos 6\theta + Cos 3\theta$

$\because Cos P + Cos Q = 2 Cos\frac{P+Q}{2} Cos\frac{P-Q}{2}$

$= 2Cos\frac{6\theta+3\theta}{2} Cos\frac{6\theta-3\theta}{2}$

$= 2Cos\frac{9\theta}{2} Cos\frac{3\theta}{2}$

2018

55. Prove that $\sin(45^0 + \alpha) = \frac{1}{\sqrt{2}}(\sin\alpha + \cos\alpha)$ **(2 times)**

Sol: $\sin(45^0 + \alpha) = \frac{1}{\sqrt{2}}(\sin\alpha + \cos\alpha)$

L.H.S

$= \sin(45^0 + \alpha)$

$= \sin 45^0 \cos\alpha + \cos 45^0 \sin\alpha$

$= \frac{1}{\sqrt{2}}\cos\alpha + \frac{1}{\sqrt{2}}\sin\alpha$

$= \frac{1}{\sqrt{2}}(\cos\alpha + \sin\alpha)$ = R.H.S

56. Prove that: $\tan\left(\frac{\pi}{4} - \theta\right) + \tan\left(\frac{3\pi}{4} + \theta\right)$

$= \left(\frac{\tan \pi/4 - \tan\theta}{1 + \tan \pi/4 \tan\theta}\right) + \left(\frac{\tan 3\pi/4 + \tan\theta}{1 - \tan 3\pi/4 \tan\theta}\right)$

$= \left(\frac{1 - \tan\theta}{1 + (1)\tan\theta}\right) + \left(\frac{-1 + \tan\theta}{1 + \tan\theta}\right)$

$= 1 - 1 = 0 =$ R.H.S

57 Express sin(x + 45°) sin(x − 45°) as sum or difference.

Sol: sin(x + 45°) sin(x − 45°)

=(sinα cos45°sin45°cosα) (sinα cos45° − cosα sin45°)

$$=\left(\frac{1}{\sqrt{2}}\sin\alpha+\frac{1}{\sqrt{2}}\cos\alpha\right)\left(\frac{1}{\sqrt{2}}\sin\alpha-\frac{1}{\sqrt{2}}\cos\alpha\right)$$

$$=\frac{1}{\sqrt{2}}[(\sin\alpha+\cos\alpha)(\sin\alpha-\cos\alpha)]$$

$$=\frac{1}{\sqrt{2}}\left[\sin^2\alpha+\sin\alpha\cos\alpha-\sin\alpha\cos\alpha-\cos^2\alpha\right]$$

$$=\frac{1}{\sqrt{2}}\left[\sin^2\alpha-\cos^2\alpha\right]=\frac{1}{\sqrt{2}}\left[\sin^2\alpha-\left(1-\sin^2\alpha\right)\right]$$

$$=\frac{1}{\sqrt{2}}\left[-1+2\sin^2\alpha\right]=\frac{1}{\sqrt{2}}\left[2\sin^2\alpha-1\right]$$ Ans.

58. Prove that $\sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}}=\frac{\sin\alpha/2+\cos\alpha/2}{\sin\alpha/2-\cos\alpha/2}$

Sol: L.H.S $=\sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}}$

$$=\sqrt{\frac{\sin^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\sin^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}-2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}}$$

$$=\sqrt{\frac{\left(\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right)^2}{\left(\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}\right)^2}}=\frac{\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}}$$

= R.H.S

59. Show that $\frac{\sin 2\alpha}{1+\cos 2\alpha}=\tan\alpha$ **(2 times)**

Sol: L.H.S

$$=\frac{\sin 2\alpha}{1+\cos 2\alpha}=\frac{2\sin\alpha\cos\alpha}{2\cos^2\alpha}=\frac{\sin\alpha}{\cos\alpha}=\tan\alpha=\text{R.H.S}$$

60. Prove that $\frac{1-Cos\alpha}{Sin\alpha}=\tan\frac{\alpha}{2}$ **(4 times)**

Sol: L.H.S

$$\because\ \sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\Rightarrow \sin^2\frac{\alpha}{2}=\frac{1-\cos\alpha}{2}\Rightarrow 2\sin^2\frac{\alpha}{2}=1-\cos\alpha$$

$$and\ \sin 2\alpha=2\sin\alpha\cos\alpha$$

$$therefore,\ \sin\alpha=2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$$

$$=\frac{1-Cos\alpha}{Sin\alpha} \qquad =\frac{2Sin^2\frac{\alpha}{2}}{2Sin\frac{\alpha}{2}Cos\frac{\alpha}{2}}$$

$$=\frac{Sin\frac{\alpha}{2}}{Cos\frac{\alpha}{2}} \qquad =\tan\frac{\alpha}{2} \qquad = \text{R.H.S}$$

2019

61: Find the value of $Cos105^0$ without using calculator.

Sol: $Cos105^0 = Cos\left(60^0+45^0\right)$

$$\because Cos(\alpha+\beta) = Cos\alpha Cos\beta - Sin\alpha Sin\beta$$

$$= Cos60^0 Cos45^0 - Sin60^0 Sin45^0$$

$$=\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)-\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}$$

$$=\frac{1}{2\sqrt{2}}-\frac{\sqrt{3}}{2\sqrt{2}}=\frac{1-\sqrt{3}}{2\sqrt{2}}$$

62: Find the value of $\tan 15^0$, without using calculator.

Sol: $\tan 15^0 = \tan\left(45^0-30^0\right)$

$$\because \tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

$$=\frac{\tan 45^0-\tan 30^0}{1+\tan 45^0\tan 30^0}$$

$$=\frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}=\frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}$$

$$=\frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}=\frac{\sqrt{3}-1}{\sqrt{3}+1}$$

63: Prove that $Cos2\alpha = Cos^2\alpha - Sin^2\alpha$

Sol: We know that

$$Cos(\alpha+\beta)=Cos\alpha Cos\beta - Sin\alpha Sin\beta$$

Let $\beta=\alpha$

So $Cos(\alpha+\alpha)=Cos\alpha Cos\alpha - Sin\alpha Sin\alpha$

$$Cos2\alpha = Cos^2\alpha - Sin^2\alpha$$

Hence proved

64: **Prove that:** $Cos306^0 + Cos234^0 + Cos162^0 + Cos18^0 = 0$ **without using calculator.**

Sol: L.H.S = $Cos306^0 + Cos234^0 + Cos162^0 + Cos18^0$

$= Cos(360^0 - 54^0) + Cos(180^0 + 54^0) + Cos(180 - 18^0) + Cos18^0$

$= Cos54^0 - Cos54^0 - Cos18^0 + Cos18^0$ =0 =R.H.S

L.H.S = R.H.S

65: **Prove the identity** $\frac{Sin\alpha - Sin\beta}{Sin\alpha + Sin\beta} = \tan\frac{\alpha-\beta}{2} Cot\frac{\alpha+\beta}{2}$

Sol: L.H.S = $\frac{Sin\alpha - Sin\beta}{Sin\alpha + Sin\beta}$

$\because SinP - SinQ = 2Cos\frac{P+Q}{2} Sin\frac{P-Q}{2}$

$\because SinP + SinQ = 2Sin\frac{P+Q}{2} Cos\frac{P-Q}{2}$

$$= \frac{\cancel{2}Cos\left(\frac{\alpha+\beta}{2}\right)Sin\left(\frac{\alpha-\beta}{2}\right)}{\cancel{2}Sin\left(\frac{\alpha+\beta}{2}\right)Cos\left(\frac{\alpha-\beta}{2}\right)}$$

$$= \frac{Cos\left(\frac{\alpha+\beta}{2}\right)}{Sin\left(\frac{\alpha+\beta}{2}\right)} \cdot \frac{Sin\left(\frac{\alpha-\beta}{2}\right)}{Cos\left(\frac{\alpha-\beta}{2}\right)}$$

$$= Cot\left(\frac{\alpha+\beta}{2}\right)\tan\left(\frac{\alpha-\beta}{2}\right)$$

Or

$$= \tan\left(\frac{\alpha-\beta}{2}\right)Cot\left(\frac{\alpha+\beta}{2}\right) \quad = \text{R.H.S}$$

L.H.S = R.H.S

66: **Prove that** $\frac{Sin\theta - Cos\theta \tan\theta/2}{Cos\theta + Sin\theta \tan\theta/2} \tan\theta/2$

Sol: L.H.S $= \frac{Sin\theta - Cos\theta \tan\theta/2}{Cos\theta + Sin\theta \tan\theta/2}$

$$= \frac{Sin\theta - Cos\theta\frac{Sin\theta/2}{Cos\theta/2}}{Cos\theta + Sin\theta.\frac{Sin\theta/2}{Cos\theta/2}}$$ Taking L.C.M

$$= \frac{\frac{Sin\theta Cos\theta/2 - Cos\theta Sin\theta/2}{\cancel{Cos}\,\theta/2}}{\frac{Cos\theta Cos\theta/2 + Sin\theta Sin\theta/2}{\cancel{Cos}\,\theta/2}}$$

$$=\frac{\sin\theta\cos\theta/2-\cos\theta\sin\theta/2}{\cos\theta\cos\theta/2+\sin\theta\sin\theta/2}$$

$\because\ \sin\alpha\cos\beta-\cos\alpha\sin\beta=\sin(\alpha-\beta)$

and $\cos\alpha\cos\beta+\sin\alpha\sin\beta=\cos(\alpha-\beta)$

$$=\frac{Sin\left(\theta-\theta/2\right)}{Cos\left(\theta-\theta/2\right)}=\frac{Sin\left(\frac{2\theta-\theta}{2}\right)}{Cos\left(\frac{2\theta-\theta}{\theta}\right)}=\frac{Sin\theta/2}{Cos\theta/2}=\tan\theta/2$$

$= R.H.S$

L.H.S = R.H.S (Hence proved)

67: Show that $\frac{Sin3\theta}{Cos\theta}+\frac{Cos3\theta}{Sin\theta}=2Cot2\theta$

Sol: L.H.S $=\frac{Sin3\theta}{Cos\theta}+\frac{Cos3\theta}{Sin\theta}$ Taking L.C.M

$$=\frac{Sin3\theta Sin\theta+Cos3\theta Cos\theta}{Sin\theta Cos\theta}$$

$\because\ \cos\alpha\cos\beta+\sin\alpha\sin\beta=\cos(\alpha-\beta)$

$$=\frac{Cos(3\theta-\theta)}{Sin\theta Cos\theta}=\frac{Cos2\theta}{Sin\theta Cos\theta}$$

$$=\frac{2Cos2\theta}{2Sin\theta Cos\theta}=2\frac{Cos2\theta}{Sin2\theta}$$

$=2Cot2\theta=R.H.S$

L.H.S = R.H.S (Hence Proved)

68: Express $Sin12^0\ Sin46^0$ **as sum or difference**

Sol: $Sin12^0\ Sin46^0$

$=Sin46^0\ Sin12^0$

Multiplying and dividing by -2

$$=\frac{-1}{-2}\left(-2Sin46^0Sin12^0\right)$$

$\because -2Sin\alpha Sin\beta=Cos(\alpha+\beta)-Cos(\alpha-\beta)$

$$=-\frac{1}{2}\left[Cos\left(46^0+12^0\right)-Cos\left(46^0-12^0\right)\right]$$

$$=-\frac{1}{2}\left(Cos58^0-Cos34^0\right)$$ Which is required

69: Show that $\frac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta}=\frac{Sin(\alpha+\beta)}{Sin(\alpha-\beta)}$

Sol: L.H.S $=\frac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta}$

$$=\frac{\frac{Sin\alpha}{Cos\alpha}+\frac{Sin\beta}{Cos\beta}}{\frac{Sin\alpha}{Cos\alpha}-\frac{Sin\beta}{Cos\beta}}$$

$$=\frac{\frac{Sin\alpha Cos\beta+Cos\alpha Sin\beta}{Cos\alpha Cos\beta}}{\frac{Sin\alpha Cos\beta-Cos\alpha Sin\beta}{Cos\alpha Cos\beta}}$$

$\because\ \sin\alpha\cos\beta-\cos\alpha\sin\beta=\sin(\alpha-\beta)$

and $\cos\alpha\cos\beta+\sin\alpha\sin\beta=\cos(\alpha-\beta)$

$$=\frac{Sin\alpha Cos\beta+Cos\alpha Sin\beta}{Sin\alpha Cos\beta-Cos\alpha Sin\beta}=\frac{Sin(\alpha+\beta)}{Sin(\alpha-\beta)}=R.H.S$$

L.H.S = R.H.S (Hence Proved)

70: Explain $Sin319^0$ as a trigonometric function of an angle of positive degree measure of less than 45^0. **(2 times)**

Sol: $Sin319^0$

$= Sin(360^0 - 41^0)$ $\quad \because Sin(360^0 - \theta) = -Sin\theta$

$= -Sin41^0$

71: Find the value of $Cos2\alpha$, when $Sin\alpha = \frac{12}{13}$ where $0 < \alpha < \frac{\pi}{2}$

Sol: Given $Sin\alpha = \frac{12}{13}$ and $\alpha \in I\ Quad$

$Cos2\alpha = 1 - 2Sin^2\alpha$

$= 1 - 2\left(\frac{12}{13}\right)^2 \quad = 1 - 2\left(\frac{144}{169}\right)$

$= 1 - \frac{288}{169} = \frac{169 - 288}{169}$

$Cos2\alpha = \frac{-119}{169}$

72: Prove that : $\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$

Sol: R.H.S $= \frac{2\tan\alpha}{1 - \tan^2\alpha}$

$= \frac{2\frac{Sin\alpha}{Cos\alpha}}{1 - \frac{Sin^2\alpha}{Cos^2\alpha}}$

$= \frac{2\frac{Sin\alpha}{Cos\alpha}}{\frac{Cos^2\alpha - Sin^2\alpha}{Cos^2\alpha}}$ $\quad \because Cos2\alpha = Cos^2\alpha - Sin^2\alpha$

$= \frac{2\frac{Sin\alpha}{Cos\alpha}}{\frac{Cos2\alpha}{Cos^2\alpha}} = 2\frac{Sin\alpha}{\cancel{Cos\alpha}} \times \frac{\cos^{\cancel{2}}\alpha}{Cos2\alpha}$

$= \frac{2Sin\alpha Cos\alpha}{Cos2\alpha}$ $\quad \because Sin2\alpha = 2Sin\alpha Cos\alpha$

$= \frac{Sin2\alpha}{Cos2\alpha} = \tan 2\alpha$

$= L.H.S$

L.H.S = R.H.S (Hence Proved)

73: Prove that $\frac{\tan\theta/2 + Cot\theta/2}{Cot\theta/2 - \tan\theta/2} = Sec\theta$

Sol: L.H.S $= \frac{\tan\theta/2 + Cot\theta/2}{Cot\theta/2 - \tan\theta/2}$

$$=\frac{\frac{Sin\theta/2}{Cos\theta/2}+\frac{Cos\theta/2}{Sin\theta/2}}{\frac{Cos\theta/2}{Sin\theta/2}-\frac{Sin\theta/2}{Cos\theta/2}}$$ Taking L.C.M

$$=\frac{\frac{Sin^2\theta/2+Cos^2\theta/2}{Cos\theta/2\,Sin\theta/2}}{\frac{Cos^2\theta/2-Sin^2\theta/2}{Cos\theta/2\,Sin\theta/2}}$$

$$=\frac{Sin^2\theta/2+Cos^2\theta/2}{Cos^2\theta/2-Sin^2\theta/2}$$ $\because Sin^2\theta/2+Cos^2\theta/2=1$

$\because \cos 2\theta=\cos^2\theta-\sin^2\theta$

$$=\frac{1}{Cos\theta}$$ $\because Cos\theta=Cos^2\theta/2-Sin^2\theta/2$

$=Sec\theta$ =R.H.S

L.H.S = R.H.S (Hence Proved)

2021

74- Express cos12° + cos 48° as product.

Sol: $\cos 12^\circ+\cos 48^\circ=2\cos\left(\frac{12^\circ+48^\circ}{2}\right)\cos\left(\frac{12^\circ-48^\circ}{2}\right)$

$\because \cos P+Cos\theta=2Cos\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$

$=2\cos 30^\circ\cos(-18)$

$=2\cos 30^\circ\cos 18^\circ$ $\because \cos(-\theta)=\cos\theta$

75- If α,β,γ are angles of triangle ABC then prove that $\cos(\alpha+\beta)=-\cos\gamma$

Sol: Let α,β,γ are the angles of a triangle A B C

So, $\alpha+\beta+\gamma=180^\circ$

$\alpha+\beta=180^\circ-\gamma$

Taking cos on both sides.

$\cos(\alpha+\beta)=\cos(180^\circ-\gamma)$

$\cos(\alpha+\beta)=-\cos\gamma$ $\because \cos(180^\circ-\theta)=-\cos\gamma$

Hence proved it.

76- Prove that $\frac{\sin 8x+\sin 2x}{\cos 8x+\cos 2x}=\tan 5x$

Sol: $L.H.S=\frac{\sin 8x+\sin 2x}{\cos 8x+\cos 2x}$

$$=\frac{2\sin\left(\frac{8x+2x}{2}\right)\cos\left(\frac{8x-2x}{2}\right)}{2\cos\left(\frac{8x+2x}{2}\right)\cos\left(\frac{8x-2x}{2}\right)}$$

$$=\frac{2\sin 5x.\cos 3x}{2\cos 5x.\cos 3x} \quad =\frac{\sin 5x}{\cos 5x}=\tan 5x = R.H.S$$

Hence L.H.S = R.H.S

77- Express the difference sin 8 θ - sin 4 θ as product

Sol: $\sin 8\theta-\sin 4\theta=2\cos\left(\frac{8\theta+4\theta}{2}\right)\sin\left(\frac{8\theta-4\theta}{2}\right)$

$$\because \sin P-\sin Q=2\cos\left(\frac{P+Q}{2}\right)Sin\left(\frac{P-Q}{2}\right)$$

$$=2\cos\left(\frac{12\theta}{2}\right)\sin\left(\frac{4\theta}{2}\right)$$

$$=2\cos 6\theta+\sin 2\theta$$

78- Find the value of tan (111 0)° **(2 Times)**

Sol: $\tan(1110^o)=\tan(12\times 90^o+30^o)$

$=\tan 30^o \qquad \because 1110^o lies\ I-Quad$

$$=\frac{1}{\sqrt{3}}$$

79- Show that $\cot(\alpha-\beta)=\frac{\cot\alpha\cot\beta+1}{\cot\beta-\cot\alpha}$ **(2 Times)**

Sol: $R.H.S=\frac{\cot\alpha\cot\beta+1}{\cot\beta-\cot\alpha}$

$$=\frac{\frac{\cos\alpha}{\sin\alpha}.\frac{\cos\beta}{\sin\beta}+1}{\frac{\cos\beta}{\sin\beta}-\frac{\cos\alpha}{\sin\alpha}}$$

$$=\frac{\frac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\sin\alpha\sin\beta}}{\frac{\sin\alpha\cos\beta-\cos\alpha\sin\beta}{\sin\alpha\sin\beta}}$$

$$=\frac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\sin\alpha\cos\beta-\cos\alpha\sin\beta} \quad =\frac{\cos(\alpha-\beta)}{\sin(\alpha-\beta)}$$

$$=\cot(\alpha-\beta) \quad \because \cot\alpha=\frac{\cos\alpha}{\sin\alpha} = L.H.S$$

L.H.S = R.H.S

80- Without using tables write down the value of cos 315°

Sol: $\cos 135^o=\cos(270^o+45^o)$

$=\sin 45^o \qquad \because \cos(270^o+\theta)=\sin\theta$

$$\frac{1}{\sqrt{2}}$$

81- Prove that $\frac{\sin A+\sin 2A}{1+\cos A+\cos 2A}=\tan A$ **(2 Times)**

Sol: $L.H.S=\frac{SinA+Sin2A}{1+\cos A+\cos 2A}$

$= \frac{SinA + Sin2A\cos A}{1+\cos A + 2\cos^2 A - 1}$

$= \frac{SinA(1+2\cos A)}{\cos A(1+2\cos A)}$

$= \frac{SinA}{\cos A} = \tan A$

= R.H.S

Hence L.H.S = R.H.S

82- Prove that $\sin\left(\theta+\frac{\pi}{6}\right)+\cos\left(\theta+\frac{\pi}{3}\right)=\cos\theta$

Sol: $L.H.S = \sin\left(\theta+\frac{\pi}{6}\right)+\cos\left(\theta+\frac{\pi}{3}\right)$

$= \sin(\theta+30^o)+\cos(\theta+60^o)$ $\because \frac{\pi}{6}=30^o$ $\because \frac{\pi}{3}=60^o$

$= \sin\theta\cos 30^o + \cos\theta\sin 30^o + \cos\theta\cos 60^o - \sin\theta\sin 60^0$

$= \sin\theta.\left(\frac{\sqrt{3}}{2}\right)+\cos\theta.\left(\frac{1}{2}\right)+\cos\theta\left(\frac{1}{2}\right)-\sin\theta\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\cos\theta}{2}+\frac{\cos\theta}{2}$

$= \frac{\cos\theta+\cos\theta}{2}$

$= \frac{2\cos\theta}{2} = \cos\theta = R.H.S$

Hence L.H.S = R.H.S

83- Express 2 sin7θ sin 2θ as sums or differences.

Sol: $2\sin 7\theta\sin 2\theta$

$= -(-2\sin 7\theta\sin 2\theta)$

$= -[\cos(7\theta+2\theta)-\cos(7\theta-2\theta)]$

$\because -2\sin\alpha\sin\beta = \cos(\alpha+\beta)-\cos(\alpha-\beta)$

$= -[\cos 9\theta - \cos 5\theta]$

$= -\cos 9\theta + \cos 5\theta$

$= \cos 5\theta - \cos 9\theta$

84- Prove that $\cot\alpha - \tan\alpha = 2\cot 2\alpha$ **(4 Times)**

Sol: L.H.S = $\cot\alpha - \tan\alpha$

$= \frac{\cos\alpha}{\sin\alpha}-\frac{\sin\alpha}{\cos\alpha}$

$= \frac{\cos^2\alpha - \sin^2\alpha}{\sin\alpha.\cos\alpha}$

$= \frac{\cos 2\alpha}{\sin\alpha.\cos\alpha}$ $\because \cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$= \frac{2\cos 2\alpha}{2\sin\alpha.\cos\alpha}$ Multiplying up and down by '2'

$$= \frac{2\cos 2\alpha}{\sin 2\alpha}$$
$$= 2\cot 2\alpha$$
= R.H.S

Hence L.H.S = R.H.S

LONG QUESTION'S OF CHAPTER-10 IN ALL PUNJAB BOARDS 2011-2021

Topic I: Fundamental Law of Trignometric:

1. If α, β, γ are the angles of ΔABC, prove that. $\text{Tan}\frac{\alpha}{2}.\text{Tan}\frac{\beta}{2} + \text{Tan}\frac{\beta}{2}.\text{ Tan}\frac{\gamma}{2} + \text{Tan}\frac{\gamma}{2}.\text{ Tan}\frac{\alpha}{2} = 1$
2. Prove that $\frac{\cos(90^o+\theta).\sec(-\theta).\tan(180^o-\theta)}{\sec(360^o-\theta).\sin(180^o+\theta).\cot(90^o-\theta)} = -1$

Topic II: Half, Double and Tripple angle Indenties:

3. Prove that $\frac{\sin 3\theta}{\cos\theta} + \frac{\cos 3\theta}{\sin\theta} = 2\cot 2\theta$ (2 times)
4. Prove the given Identity: $\frac{\tan\frac{\theta}{2} + \cot\frac{\theta}{2}}{\cot\frac{\theta}{2} - \tan\frac{\theta}{2}} = \sec\theta$
5. Prove the identity $\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2$
6. Prove that $\frac{\cos 3\theta}{\cos\theta} + \frac{\sin 3\theta}{\sin\theta} = 4\cos 2\theta$ **(3 times)**
7. Reduce $\cos^4\theta$ to an expression involving only function of multiples of θ raised to first power.

Topic III: Sum, Difference and Product Sines and Cosines:

8. Prove that without using calculator/table $\sin\frac{\pi}{9}\sin\frac{2\pi}{9}\sin\frac{\pi}{3}\sin\frac{4\pi}{9} = \frac{3}{16}$. (2 times)
9. Prove that $\cos 20^\circ\cos 40^\circ\cos 60^\circ\cos 80^\circ = \frac{1}{16}$ (4 times)
10. Prove that $\sin 10^\circ\sin 30^\circ\sin 50^\circ\sin 70^\circ = \frac{1}{16}$ (2 times)
11. Prove that $\frac{\sin\theta+\sin 3\theta+\sin 5\theta+\sin 7\theta}{\cos\theta+\cos 3\theta+\cos 5\theta+\cos 7\theta} = \tan 4\theta$ **(3 times)**
12. Prove that: $\frac{\sin\alpha-\sin\beta}{\sin\alpha+\sin\beta} = \tan\frac{\alpha-\beta}{2}\cot\frac{\alpha+\beta}{2}$
13. Prove that $\frac{2\sin\theta\sin(2\theta)}{\cos\theta+\cos(3\theta)} = \tan(2\theta)\tan\theta$
14. Prove that $\frac{2\sin\theta\sin 2\theta}{\cos\theta+\cos 3\theta} = \tan 2\theta\tan\theta$ (3 Times)
15. If $\tan\alpha = \frac{3}{4}, \cos\beta = \frac{5}{13}$ and neither the terminal side of the angle of measure α nor that of β is in the I quadrant, Find $\sin(\alpha+\beta)$
16. Prove without using calculator $\sin 19^o\cos 11^o + \sin 71^o\sin 11^o = \frac{1}{2}$
17. Prove $\frac{\cos 8^o - \sin 8^o}{\cos 8^o + \sin 8^o} = \tan 37^o$
18. Prove without using calculator $\sin 19^o\cos 11^o + \sin 71^o\sin 11^o = \frac{1}{2}$

OBJECTIVE MCQ'S OF CHAPTER-11 IN ALL PUNJAB BOARDS 2011-2021

1. Domain of y = Sec x is ____________: (2 times)
(A) $\Re-\left\{x \mid x=\frac{2n+1}{2}\pi\right\}$ (B) $-\infty \leq x \leq \infty$
(C) $-\pi \leq x \leq \pi$ (D) $-\pi < x\,\pi,\ X \neq (2n+1)\frac{\pi}{2}+n\in z$

2. Domain of cosine function is: (4 times)
(A) $\Re$ (B) Z (C) C (D) W

3. The domain of Tan x is: (5 times)
(A) $\Re$ (B) [-1 , 1]
(C) $\Re - \{x \mid x = n\pi\ ; n \in z$ (D) $\Re - \left\{x \mid x = (2n+1)\frac{\pi}{2}; n \in z\right\}$

4. Period of Tan $\frac{x}{3}$ is: (4 times)
(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) 3π

5. Period of Cos $\frac{x}{3}$ is: (4 times)
(A) π (B) π (C) $\frac{2\pi}{3}$ (D) 6π

6. Period of Cot 8x is: (6 times)
(A) 8π (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{8}$ (D) π

7. The period of 3 sin $\frac{x}{3}$ is: (8 times)
(A) π (B) 2π (C) 3π (D) 6π

8. The range of Sin x is: (2 times)
(A) [-1 ,1] (B) [-1 ,0] (C) [0,2] (D) [-2 ,2]

9. Period of tan2x is: (3 times)
(A) π (B) 2π (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

10. Period of 3cos$\frac{x}{5}$ is: (5 times)
(A) 10π (B) 3π (C) $\frac{\pi}{5}$ (D) $\frac{3\pi}{5}$

11. Range of y = cosx is equal to:- (4 times)
(A) (-1,1) (B) $-1 \leq y \leq 1$ (C) $0 < y < 1$ (D) $\Re$

12. The period of Sec x is: (5 times)
(A) π (B) 2π (C) 3π (D) 4π

13. The range of cos x is: (8 times 2018)
(A) R (B) [-1,1] (C) [-3,3] (D) [0,3]

14. The period of cot $\frac{x}{3}$ is: (4 times)
(A) π (B) 2π (C) 3π (D) 4π

15. Range of tangent function is:- (2 times)
(A) $\Re$ (B) Z (C) N (D) C

16. Period of cosec 10x is: (2 times)
(A) $\frac{\pi}{10}$ (B) $\frac{2\pi}{5}$ (C) $\frac{\pi}{5}$ (D) $\frac{4\pi}{5}$

17. The period of cot 8x is:
(A) π (B) 8π (C) $\pi/8$ (D) 2π

18. Range of the tangent function is:
(A) R (B) Z (C) $-1 < x < 1$ (D) $-1 \leq x \leq 1$

19. The period of Cosec x/3 is:
(A) π (B) 3π (C) 6π (D) 8π

20. The period of 2 cos x is:
(A) 4π (B) 2π (C) π (D) 3π

21. The period of sin x is:
(A) 0 (B) π (C) 2π (D) $\frac{\pi}{2}$

22. Range of cot x is equal to:
(A) $\Re$ (B) Q (C) Z (D) N

23. The period of sin2x is equal to:
(A) π (B) $\pi/2$ (C) $\pi/3$ (D) 2π

24. The period of 3 cos 2x is:
(A) π (B) 2π (C) 3π (D) $\frac{\pi}{2}$

25. Range of y = Cos x is:
(A) $\Re$ (B) $-1\le x\le 1$ (C) $y\ge 1$ or $y\le -1$ (D) $-1\le y\le 1$

26. Period of y = $\tan\theta$ is
(A) π (B) 2π (C) 3π (D) 4π

27. Range of Sin2x is :
(A) [-1 , 1] (B) [-2 , 2] (C) (-1 , 1) (D) (-2 , 2)

28. The range of Sin x is
(A) [-1,1] (B) [-1,0] (C) [0,2] (D) [-2,2]

29. The range of y = sin x is equal to
(A) $-1\le y\le 1$ (B) $-1 < y < 1$ (C) $-1\le x\le 1$ (D) $-1\le y < 1$

30. Period of Sin $\left(\frac{x}{3}\right)$ is:
(A) $\overline{\wedge}$ (B) $3\overline{\wedge}$ (C) $\frac{2\overline{\wedge}}{3}$ (D) $6\overline{\wedge}$

31. Period of $\tan\frac{x}{2}$ is:
(A) π (B) 2π (C) $\frac{\pi}{2}$ (D) $\frac{3\pi}{2}$

32. The domain of cos x is:
(A) [0 , 1] (B) $\Re$ (C) [-1 , 1] (D) [-1 , 0]

33. The domain of sinx equals:
(A) [-1 , 1] (B) $\Re$ (C) π (D) Q

34. Period of tan $\left(\frac{x}{7}\right)$ is:
(A) π (B) $\pi/7$ (C) 7π (D) $\pi+7$

35. Period of tan $x/3$ is:
(A) π (B) 2π (C) 3π (D) $\pi/2$

36. Period of $\cos\frac{x}{2}$ is: (2 times)
(A) π (B) 2π (C) 4π (D) $\frac{\pi}{2}$

37. Domain of Sin x is: (2 times)
(A) $\{-1,1\}$ (B) $\Re$ (C) $\Re-\{0\}$ (D) Q

38. The period of sec x is:
(A) π (B) 2π (C) 3π (D) 4π

2018

39. The smallest positive integer p for which f(p+x) = f(x) is called:
(a) Domain (b) Range (c) Co-Domain (d) Period

40. The period of 3 sin x is:
(a) 3π (b) π (c) 2π (d) $\frac{\pi}{3}$

41. Period of Sec10 x is:
(a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{5}$ (d) 2π

42. Period of sec 10x is:

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{5}$ (d) 2π

43. The smallest positive number p for which $f(x+p)=f(x)$ is called:

(a) Index (b) Domain (c) Coefficients (d) Period

2019

44. The range of y = cosx is:

(a) $-1 \le x \le 1$ (b) $-\infty < x < \infty$ (c) $-1 \le y \le 1$ (d) $-\infty < y < \infty$

45. Period of csc θ is:

(a) π (b) $-\pi$ (c) 2π (d) -2π

46. Period of $\sin\frac{x}{5}$ is equal to:

(a) 10π (b) 5π (c) 2π (d) $\frac{2\pi}{5}$

47. The period of $\sin\frac{x}{2}$ is:

(a) 2π (b) 4π (c) π (d) 3π

48. Range of cotangent function is:

(a) N (b) Z (c) R (d) C

2021

49. π is the period of

(a) $\sec\theta$ (b) $\operatorname{cosec}\theta$ (c) $\cot\theta$ (d) $\sin 3\theta$

50. Period of 2 + cos3x is:

(a) π (b) $\frac{3\pi}{2}$ (c) 2π (d) $\frac{2\pi}{3}$

51. Period of $2\operatorname{cosec}\frac{x}{4}$ is :

(a) $\frac{\pi}{2}$ (b) 4π (c) 2π (d) 8π

ANSWERS TO THE MULTIPLE CHOICE QUESTIONS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	A	D	D	D	C	D	A	C	A	B	B	B	C	A
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
C	C	A	C	B	C	A	A	A	D	A	A	A	A	D
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
B	B	B	C	C	C	B	B	D	C	C	C	D	C	C
46	47	48	49	50	51									
A	B	C	C	D	D									

SHORT QUESTION'S OF CHAPTER-11 IN ALL PUNJAB BOARDS 2011-2021

1. Write down the domain and range of Cot x. **(3 times)**

Sol. Domain of Cot x:

$$Domain = \Re - \{x \mid x \neq n\pi, n \in z\}$$

$$Range = \Re$$

2. **Find the period of Sin $\frac{x}{3}$.** **(2 times)**

Sol. Period of Sin $\frac{x}{3}$

We know that period of Sin function is 2π

$\sin(\frac{x}{3} + 2\pi) = \sin\frac{x}{3}$

$\sin\frac{1}{3}(x + 6\pi) = \sin\frac{x}{3}$

So period of Sin $\frac{x}{3}$ is 6π

3. **Find the period of Cos $\frac{x}{6}$.** **(4 times)**

Sol. $\cos\frac{x}{6} = \cos(x/6 + 2\pi) = \cos\frac{1}{6}(x + 12\pi)$

Hence period of Cos $\frac{x}{6}$ is 12π

4. **Write the Domain and Range of y = Cosec x.**

Sol. y = Cosec x

Domain: $-\infty < x < +\infty$ $x \neq n\pi$

$n \in z$

Range: $y \geq 1$ or $y \leq -1$

5. **Find the period of Cot 8x.** **(5 times)**

Sol. Cot 8x

We know that period of Cot x is π

$\cot 8x = \cot(8x + \pi)$

$= \cot 8\left(x + \frac{\pi}{8}\right)$

so, Period of Cot 8x is $\frac{\pi}{8}$

6. **Find the domain and range of y = Tanx.** **(2 times)**

Sol. Domain of tangent function = $\Re - \{x/x \neq (2n+1)\frac{\pi}{2}, n \in z\}$

And Range of tangent function = $\Re$ = Set of all real no, s.

7. **Find the period Sin $\frac{x}{5}$.** **(6 times 2018)**

Sol. We know that the period of sine function is 2π.

$\therefore \sin\left(\frac{x}{5} + 2\pi\right) = \sin x/5$

$\Rightarrow \left[\sin\frac{1}{5}(x + 10\pi)\right] = \sin x/2$

$\Rightarrow$ Period of Sin $x/5$ is 10π.

8. **Write the domain and range of y = Cosx.**

Sol. y = Cosx

Domain = $-\infty < x < +\infty$

Range = $-1 \leq y \leq 1$ Ans.

9. **Find the period of the function 3cos$\frac{x}{5}$.** **(5 times)**

Sol. $3\cos\frac{x}{5}$

We know that the period of the cosine function is 2π

$\therefore 3\cos\left(\frac{x}{5} + 2\pi\right) = 3\cos x/5$

$\Rightarrow$ $3\cos\frac{1}{5}(x + 10\pi) = 3\cos x/5$

$\Rightarrow$ Period of $3\cos x/5$ is 10π

10. **Prove that tangent is a periodic function and its period is π.**

Sol. Tangent is a periodic function and its period is π.

Proof: Suppose 'p' is the period of tangent function such that

$\tan(\theta + p) = \tan\theta \quad \forall\, \theta \in \Re$

Now put $\theta = 0$ we have

$\tan(0 + p) = \tan\theta$

$\Rightarrow \tan p = 0$

$\therefore p = 0, \pi, 2\pi, 3\pi, \ldots$

(i) If p = π then tan $(\theta + \pi)$ = tanθ which is true

As π is the smallest +ve number for which tan $(\theta + \pi) = \tan\theta$

$\therefore$ π is the period of tanθ

11. Find the period of tan4x of the function. (4 times)

Sol. tan 4x

We know the period of tangent function is π.

$\therefore \quad \tan(4x + \pi) = \tan 4x$

$\Rightarrow \quad \tan 4\left(x + \frac{\pi}{4}\right) = \tan 4x$

so period of tan4x is $\frac{\pi}{4}$

12. Find the period of cosec $x/2$.

Sol. cosec $x/2$

We know that the period of cosecant function is 2π.

$\therefore \text{cosec}\left(\frac{x}{2} + 2\pi\right) = \text{cosec}\,\frac{x}{2}$

$\Rightarrow \quad \text{cosec}\,\frac{1}{2}(x + 4\pi) = \text{cosec}\,\frac{x}{2}$

so Period of cosec $x/2$ is 4π.

13. Find the period of cot $\frac{x}{2}$. (6 times)

Sol. We know that the period of cotangent is π.

$\therefore \quad \cot\left(\frac{x}{2} + \pi\right) = \cot\frac{x}{2}$

$\Rightarrow \quad \cot\frac{1}{2}(x + 2\pi) = \cot\frac{x}{2}$

so period of cot $\frac{x}{2}$ is 2π

14. Find the domain and range of secx

Sol Domain and range of secx is

$$Domain = \Re - \left\{x \mid x \neq \frac{(2n+1)}{2}\pi\right\}$$

Range $= y \geq 1 \quad or \quad y \leq -1$

2018

15. Find the period of $\tan\frac{x}{3}$. (2 times)

Sol: $\because$ period of tangent function is π

Therefore,

$$\tan\frac{x}{3} = \tan\left(\frac{x}{3} + \pi\right)$$

$$= \tan\frac{1}{3}(x + 3\pi)$$

Period of $\tan\frac{x}{3}$ is 3π.

16. Find the period of $\tan\frac{x}{7}$. (8 times)

Sol: $\tan\frac{x}{7} = \tan\left(\frac{x}{7} + \pi\right)$

$$= \tan\frac{1}{7}(x + \pi)$$

Period of $\tan\frac{x}{7} = 7\pi$

2019

17: Find the period of $3Sin\frac{2x}{5}$

Sol: $3Sin\frac{2x}{5}$

We know that period of $Sinx$ is 2π

$$3\sin\left(\frac{2x}{5}\right) = \sin\left(\frac{2x}{5}+2\pi\right)$$

$$= 3Sin\left(\frac{2x+10\pi}{5}\right)$$

$$= 3Sin\frac{2}{5}(x+5\pi)$$

Hence period of $3Sin\frac{2x}{5}$ is 5π

18: Find the period of $Co\sec(10x)$

Sol: $Co\sec(10x)$

We know that period of $Co\sec x$ is 2π

$\cos ec10x = (10x+2\pi)$

$= Co\sec 10\left(x+\frac{2\pi}{10}\right) \quad = Co\sec 10\left(x+\frac{\pi}{5}\right)$

Hence period of $Co\sec(10x)$ is $\frac{\pi}{5}$

19: Find the period of $Cos2x$ **(3 times)**

Sol: $Cos2x$

We know that period of $Cosx$ is 2π

$$Cos2x = Cos(2x+2\pi)$$

$$= Cos2(x+\pi)$$

Hence period of $Cos2x$ is π.

20: Find the period of $Sin3x$

Sol: $Sin3x$

We know that period of $Sinx$ is 2π

$$Sin3x = Sin(3x+2\pi)$$

$$= Sin3\left(x+\frac{2\pi}{3}\right)$$

Hence period of $Sin3x$ is $\frac{2\pi}{3}$

21: Find the period of $\tan x$

Sol: $\tan x$

We know that period of $\tan x$ is π.

$= \tan(x+\pi) = \tan x$

So period of $\tan x$ is π.

OBJECTIVE MCQ'S OF CHAPTER-12 IN ALL PUNJAB BOARDS 2011-2021

Topic I: Trignometric Ratios:

1. In a right triangle, no angle is greater than: (4 times)
(A) 90° (B) 80° (C) 60° (D) 45°

2. The sum of the three angles of a triangle is:
(A) 360° (B) 270° (C) 180° (D) 90°

3. If $\gamma = 90°$, then with usual notation : (2 times 2018)
(A) $c^2 = a^2 + b^2$ (B) $b^2 = a^2 + c^2$ (C) $a^2 = b^2 + c^2$ (D) $a^2 - b^2 = c^2$

Topic II: Solution of Oblique Triangles:

4. If ABC be a right angle triangle then the law of Cosines reduces to:
(A) The law of Sin (B) The law of Tangents (C) The pythagora's theorem (D) $c^2 + a^2 = b^2$

5. $\cos\frac{\alpha}{2}$ =: (2 times)
(A) $\sqrt{\frac{s-a}{bc}}$ (B) $\sqrt{\frac{s(s-a)}{bc}}$ (C) $\sqrt{\frac{s(s-a)}{ac}}$ (D) $\sqrt{\frac{s-b}{bc}}$

6. For a triangle ABC with usual notations $\sqrt{\frac{(S-a)(S-b)}{S(S-c)}}$ equal. (3 times)
(A) $\tan\gamma$ (B) $\tan\frac{\gamma}{2}$ (C) $\cot\gamma$ (D) $\cot\frac{\gamma}{2}$

7. $\sin\frac{\alpha}{2} =$ (3 times 2018)
(A) $\sqrt{\frac{(s-a)(s-b)}{2ab}}$ (B) $\sqrt{\frac{(s-b)(s-c)}{bc}}$ (C) $\sqrt{\frac{(s-b)(s-a)}{bc}}$ (D) $\sqrt{\frac{(s+b)(s+c)}{bc}}$

8. In any triangle ABC, a^2 is: (1 time)
(A) $b^2 + c^2 - 2bc\cos\alpha$ (B) $c^2 + a^2 - 2ca\cos\beta$ (C) $a^2 - b^2 - 2ab\cos\gamma$ (D) $b^2 + c^2 + 2bc\cos\alpha$

9. In any Δ ABC, $\frac{c^2+a^2-b^2}{2ca} =$
(A) $\cos\beta$ (B) $\cos\alpha$ (C) $\cos\gamma$ (D) $\cos\frac{\beta}{2}$

10. $\cot\frac{\alpha}{2}$ equal to: (2 times)
(A) $\sqrt{\frac{s(s-c)}{(s-b)(s-a)}}$ (B) $\sqrt{\frac{s(s-a)}{(s-b)(s-c)}}$ (C) $\sqrt{\frac{(s-b)(s-a)}{s(s-c)}}$ (D) $\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

11. $\sqrt{\frac{s(s-a)}{bc}}$ = : (2 times)
(A) $\sin\frac{\alpha}{2}$ (B) $\sin\frac{\beta}{2}$ (C) $\cos\frac{\alpha}{2}$ (D) $\cos\frac{\beta}{2}$

12. If $\beta = 90°$ then law of cosine:
(A) Law of sines (B) Law of tangents (C) Pythagoras theorem (D) Fundamental law

13. With usual notations $\sqrt{\frac{(s-b)(s-c)}{s(s-a)}} =$
(A) $\tan\frac{\alpha}{2}$ (B) $\tan\frac{\beta}{2}$ (C) $\tan\frac{\gamma}{2}$ (D) $\cot\frac{\alpha}{2}$

14. In any triangle with usual notation $\sqrt{\frac{(S-c)(S-a)}{S(S-b)}} =$
(A) $\cos\frac{\beta}{2}$ (B) $\sin\frac{\beta}{2}$ (C) $\tan\frac{\beta}{2}$ (D) $\cot\frac{\beta}{2}$

15. $\tan\frac{\alpha}{2} =$
(A) $\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ (B) $\sqrt{\frac{s(s-b)}{ac}}$ (C) $\sqrt{\frac{(s-a)(s-b)}{ab}}$ (D) $\sqrt{\frac{(s-a)(s-c)}{bc}}$

16. $\sqrt{\frac{s(s-a)}{bc}}$ equal:

(A) $\sin\frac{\alpha}{2}$ (B) $\sin\frac{\beta}{2}$ (C) $\cos\frac{\alpha}{2}$ (D) $\cos\frac{\beta}{2}$

17. With usual notation the value of a + b – c equals:

(A) 2 (s – c) (B) 2 (s – b) (C) 2 (s – a) (D) 2s

18. $\tan\frac{\alpha}{2}$ =

(A) $\sqrt{\frac{s(s-a)}{(s-b)(s-c)}}$ (B) $\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$ (C) $\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ (D) $\frac{s}{\Delta}$

19. $b^2 + c^2 - 2bc\cos\alpha$ = :

(A) Δ (B) 0 (C) a^2 (D) 1

Topic III: Area of Triangle:

20. In a Triangle ABC, Δ = 20 a = 4 b = 6 c = 10 then r = ______

(A) 2 (B) 5 (C) 10 (D) 15

21. Area of ΔABC = **(4 times)**

(A) $\frac{1}{2}ab\sin\alpha$ (B) $ab\sin\gamma$ (C) $\frac{1}{2}ac\sin\beta$ (D) $ab\sin\alpha$

22. $ab\sin\gamma$ = **(3 times)**

(A) Area of triangle ABC (B) $\frac{1}{2}$ (area of triangle ABC)

(C) 2 (area of triangle ABC) (D) 3(are of triangle ABC)

23. If Δ is the area of a triangle ABC, then Δ = :

(A) $\frac{1}{2}ab\sin\gamma$ (B) $\frac{1}{2}ab\sin\beta$ (C) $\frac{1}{2}ab\cos\gamma$ (D) $\frac{1}{2}ab\cos\beta$

24. Area of triangle ABC is equal to:

(A) $\frac{1}{2}ab\sin\beta$ (B) $\frac{1}{2}bc\sin\alpha$ (C) $\frac{1}{2}ac\sin\gamma$ (D) $\frac{1}{2}ab\sin\alpha$

25. With usual notations area of triangle ABC is:

(A) $\frac{b}{2\sin\beta}$ (B) $\frac{b}{2\sin\beta}$ (C) $\frac{ac\cos\beta}{2}$ (D) ½ ab Sin γ

Topic IV: Circles Connected with Triangle:

26. $\frac{\Delta}{s}$ =

(A) R (B) r_1 (C) r (D) r_2

27. In – radius r =: **(3 times)**

(A) $\frac{\Delta}{s-a}$ (B) $\frac{\Delta}{s-b}$ (C) $\frac{\Delta}{s}$ (D) $\frac{\Delta}{s-c}$

28. Radius of escribed circle opposite to vertex B is: **(7 times)**

(A) $\frac{\Delta}{s}$ (B) $\frac{\Delta}{s-a}$ (C) $\frac{\Delta}{b}$ (D) $\frac{\Delta}{s-b}$

29. The circum – radius R is equal to:- **(4 times)**

(A) $\frac{a+b+c}{4\Delta}$ (B) $\frac{a+b+c}{\Delta}$ (C) $\frac{abc}{\Delta}$ (D) $\frac{abc}{4\Delta}$

30. E-Radius r_1 = **(7 times)**

(A) $\frac{\Delta}{s-a}$ (B) $\frac{s-a}{\Delta}$ (C) $\frac{\Delta}{s}$ (D) $\frac{s}{\Delta}$

31. With usual notation $\frac{abc}{4\Delta}$ is equal to: **(2 times)**

(A) r (B) 2r (C) R (D) r_1

32. r_2 = **(2 times)**

(A) $\frac{\Delta}{s-a}$ (B) $\frac{\Delta}{s-b}$ (C) $\frac{a}{b}$ (D) $\frac{b}{a}$

33. For any triangle ABC, with usual notations r_2 is equal to: **(2 times)**

(A) $\frac{\Delta}{s-a}$ (B) $\frac{\Delta}{s-c}$ (C) $\frac{\Delta}{s-b}$ (D) $\frac{\Delta}{s}$

34. Notation of Radius of In-Circle is:
(A) r (B) R (C) r_1 (D) Δ

35. Circum radius R is equal to: (2 times)
(A) $\frac{abc}{\Delta}$ (B) $\frac{4abc}{\Delta}$ (C) $\frac{abc}{4\Delta}$ (D) $\frac{\Delta}{s}$

36. Circus radius R is given by:
(A) $\frac{b}{2\, Sin\gamma}$ (B) $\frac{a}{2\, Sin\, \beta}$ (C) $\frac{c}{2\, Sin \propto}$ (D) $\frac{b}{2\, Sin\, \beta}$

37. With usual notation $r:R:r_1$ =
(A) 1:2:3 (B) 3:2:1 (C) 2:1:3 (D) 2:3:1

38. Radius of the inscribed circle is: (2 times)
(A) $r = \frac{\Delta}{s}$ (B) $r = \frac{abc}{4\Delta}$ (C) $r = \frac{s}{\Delta}$ (D) $r = \frac{s-a}{\Delta}$

39. With usual notations R, (4 times)
(A) $\frac{abc}{4\Delta}$ (B) $\frac{abc}{\Delta}$ (C) $\frac{4abc}{\Delta}$ (D) $\frac{4\Delta}{abc}$

40. Radius of escribed circle opposite to the vertex C is equal to: (2 times)
(A) $\frac{\Delta}{s-a}$ (B) $\frac{\Delta}{s-b}$ (C) $\frac{\Delta}{s-c}$ (D) $\frac{\Delta}{s}$

41. R is equal to:
(A) $\frac{\Delta}{abc}$ (B) $\frac{abc}{\Delta}$ (C) $\frac{a}{2\sin a}$ (D) $\frac{\Delta}{s}$

42. r_1 is equal to:
(A) $\frac{\Delta}{s-a}$ (B) $\frac{\Delta}{s-b}$ (C) $\frac{\Delta}{s-c}$ (D) $\frac{s-a}{\Delta}$

43. The radius of inscribed circle is: (2 times 2018)
(A) $\frac{abc}{4\Delta}$ (B) $\frac{\Delta}{s}$ (C) $\frac{\Delta}{s-a}$ (D) $\frac{s}{\Delta}$

44. For any triangle ABC, with usual notations r_1 is equal to:
(A) $\frac{\Delta}{s-b}$ (B) $\frac{\Delta}{s-a}$ (C) $\frac{s-a}{\Delta}$ (D) $\frac{\Delta}{s-c}$

45. R =
(A) $\frac{abc}{\Delta}$ (B) $\frac{4abc}{\Delta}$ (C) $\frac{abc}{4\Delta}$ (D) $\frac{4\Delta}{abc}$

46. R is equal to:
(A) $\frac{\Delta}{abc}$ (B) $\frac{abc}{4\Delta}$ (C) $\frac{b}{2Sin\beta}$ (D) $\frac{\Delta}{s}$

47. Notation for radius of in-circle is:
(A) r (B) R (C) r_1 (D) Δ

48. The point of intersection of the angle bisectors of a triangle is called:
(A) Circum centre (B) Orthocentre (C) In-centre (D) Centroid

49. Radius of escribed circle opposite to vertex A is equal to:
(A) $\frac{\Delta}{s}$ (B) $\frac{\Delta}{s-b}$ (C) $\frac{\Delta}{s-c}$ (D) $\frac{\Delta}{s-a}$

50. e – radius corresponding to < B equals:
(A) $\frac{\Delta}{s-a}$ (B) $\frac{\Delta}{s-b}$ (C) $\frac{\Delta}{s-c}$ (D) $\frac{\Delta}{s}$

51. In an equilateral triangle with usual notations $r : R : r_1$ is:
(A) 1 : 1 : 1 (B) 2 : 1 : 2 (C) 3 : 2 : 1 (D) 1 : 2 : 3

52. Radius of escribed circle is:
(A) $r_1 = \frac{\Delta}{s-a}$ (B) $r_1 = \frac{\Delta}{a}$ (C) $r_1 = \frac{s-a}{\Delta}$ (D) $r_1 = \frac{s}{\Delta}$

53. Radius of inscribed circle is:
(A) $\frac{\Delta}{S}$ (B) $\frac{S}{\Delta}$ (C) $\frac{\Delta}{S-c}$ (D) $\frac{4\Delta}{abc}$

54. Wit usual notations, r_1 equals: (2 times)
(A) $\frac{\Delta}{S}$ (B) $\frac{\Delta}{S-a}$ (C) $\frac{\Delta}{S-b}$ (D) $\frac{\Delta}{S-c}$

55. With usual notations, in any triangle ABC, if $\Delta = 20$ a = 4, b = 6, c = 10, then r equals:

(A) 2 (B) 5 (C) 10 (D) 15

56. In an equilateral ΔABC, r : R=

(A) 2 : 3 (B) 1 : 3 (C) 3 : 2 (D) 1 : 2

57. With usual notation $\frac{abc}{4\Delta} =$:

(A) r (B) 2r (C) R (D) r_1

2018

58. With usual notation in triangle ΔABC, if a = 7, b = 3, c = 5 the value of "S" is equal to.

(a)15 (b) $\frac{15}{2}$ (c)55 (d) 105

59. If ΔABC is right angle triangle, the law of cosine reduces to the.

(a) Law of Sine (b) Area of triangle (c) Law of tangent (d) Pythagoras Theorem

60. In any triangle ABC, with usual notation $\tan\frac{8}{2} =$

(a) $\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$ (b) $\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ (c) $\sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$ (d) $\sqrt{\frac{s(s-a)}{bc}}$

61. In any triangle ABC, with usual notation $\tan\frac{r}{2}$

(a) $\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$ (b) $\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ (c) $\sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$ (d) $\sqrt{\frac{s(s-a)}{bc}}$

62. A tree of 8 meters high has shadow 8 m in length, then angle of elevation of sun at the moment is:

(a) 15° (b) 30° (c)45° (d) 60°

63. In a triangle with usual notation $\cos\frac{\beta}{2} =$

(a) $\sqrt{\frac{(s-a)(s-c)}{ac}}$ (b) $\sqrt{\frac{(s-a)(s-b)}{ab}}$ (c) $\sqrt{\frac{s(s-c)}{ab}}$ (d) $\sqrt{\frac{s(s-b)}{ac}}$

2019

64. $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$ is called the law of:

(a) tangents (b) cosine (c) sines (d) cotangents

65. If Δ is the area of a triangle ABC then Δ =:

(a) $\frac{1}{2}bc\sin\beta$ (b) $\frac{1}{2}ab\sin\alpha$ (c) $\frac{1}{2}bc\sin\alpha$ (d) $ab\sin\alpha$

66. Angle below the horizontal ray is called:

(a) Right angle (b) Oblique angle (c) angle of depression (d) Angle of elevation

67. With usual notation, $\gamma_1 =$:

(a) $\frac{\Delta}{s-b}$ (b) $\frac{\Delta}{s-a}$ (c) $\frac{\Delta}{s-c}$ (d) $\frac{s-a}{\Delta}$

68. The angle above the Horizontal line is called an angle of:

(a) Depression (b) Elevation (c) Allied (d) Quadrantal

69. Radius of e – circle opposite to vertex "A" of ΔABC is:

(a) $\frac{\Delta}{s}$ (b) $\frac{\Delta}{s-a}$ (c) $\frac{\Delta}{s-b}$ (d) $\frac{\Delta}{s-c}$

70. Area of ΔABC in terms of measure of its all sides is:

(a) $\frac{1}{2}bc\sin\alpha$ (b) $\frac{c^2\sin\alpha\sin\beta}{2\sin\gamma}$ (c) $\frac{1}{2}ca\sin\beta$ (d) $\sqrt{s(s-a)(s-b)(s-c)}$

71. In an oblique triangle, if a = 200; b = 120 and included angle $\gamma = 150^0$, then its area will be equal to:

(a) 6000 (b) 5000 (c) 2000 (d) 12000

72. In any triangle ABC, with usual notation $b^2+c^2-2bc\cos\alpha =$

(a) Δ (b) 0 (c) a^2 (d) 1

73. $\sqrt{\frac{s(s-a)}{bc}} =$:

(a) $\sin\alpha/2$ (b) $\sin\beta/2$ (c) $\cos\alpha/2$ (d) $\cos\beta/2$

74. With usual notation r_3 = :

(a) $\frac{\Delta}{s-b}$ (b) $\frac{\Delta}{s-a}$ (c) $\frac{\Delta}{s-c}$ (d) $\Delta^2(s-c)$

75. If Δ is the area of a triangle ABC, then with usual notation Δ =:

(a) $\frac{1}{2}bc\sin\beta$ (b) $\frac{1}{2}bc\sin\beta$ (c) $\frac{1}{3}bc\sin\alpha$ (d) $\frac{1}{2}bc\sin\alpha$

2021

76. If a,b,c have their usual meanings then $\frac{c^2+a^2-b^2}{2ac} =$

(a) $\cos\alpha$ (b) $\cos\beta$ (c) $\cos\gamma$ (d) $\sin\beta$

77. In any triangle ABC, with usual notation $\sqrt{\frac{s(s-c)}{ab}} =$

(a) $\cos\frac{\gamma}{2}$ (b) $\cos\frac{\alpha}{2}$ (c) $\cos\frac{\beta}{2}$ (d) $\sin\frac{\alpha}{2}$

78. Radius of e-circle opposite to vertex 'A' of ΔABC is:

(a) $\frac{\Delta}{s-a}$ (b) $\frac{\Delta}{s-c}$ (c) $\frac{\Delta}{s}$ (d) $\frac{\Delta}{s-b}$

79. $\frac{C^2\sin\alpha\sin\beta}{\sin\gamma} =$

(a) Δ (b) $\frac{\Delta}{2}$ (c) 2Δ (d) ΔS

80. If any triangle ABC with usual notations $\frac{b^2+c^2-a^2}{2bc}$ equal to

(a) $\cos\beta$ (b) $\cos\alpha$ (c) $\sin\beta$ (d) $\sin\alpha$

81. If the sides of a triangle are 18,24,30 then the value of S is

(a) 36 (b) 72 (c) 144 (d) 24

82. If α,β and γ are the angles of an ablique Triangle, then it must be true that

(a) $\alpha = 90''$ (b) $\beta = 90^o$ (c) $\gamma = 90^o$ (d) No angle is 90^o

83. In any Triangle ABC, with usual notations, $\frac{a}{2\sin\alpha} =$

(a) Δ (b) r (c) 2R (d) R

84. In any Triangle ABC, with usual notation $\frac{b-c}{b+c}=$

(a) $\frac{\tan\frac{\beta-\gamma}{2}}{\tan\frac{\beta+\gamma}{2}}$ (b) $\frac{\tan\frac{\beta+\gamma}{2}}{\tan\frac{\beta-\gamma}{2}}$ (c) $\frac{\tan\frac{\alpha-\gamma}{2}}{\tan\frac{\alpha+\gamma}{2}}$ (d) $\frac{\tan\frac{\alpha+\beta}{2}}{\tan\left(\frac{\alpha-\beta}{2}\right)}$

85. In any triangle ABC, a = 4, b = 10, $\gamma = 30''$, then area of triangle $\gamma = 30''$ = :

(a) 5 sq.units (b) 10 sq.units (c) 20 sq.units (d) 40 sq.units

86. If r is the radius and C is the circumference of a circle, then value of $\frac{C}{r}$ =:

(a) π (b) $\frac{\pi}{2}$ (c) 2π (d) $\frac{1}{2\pi}$

87. With usual notations, $\frac{abc}{\Delta}$ =:

(a) 4R (b) r (c) R (d) rs

88. In any triangle ABC, if two sides and their included angle is given, then area of triangle is:

(a) $\Delta=\frac{1}{2}bc\sin\alpha$ (b) $\frac{1}{2}ab\sin\gamma$ (c) $\Delta=\frac{1}{2}ac\sin\beta$ (d) All these

ANSWERS TO THE MULTIPLE CHOICE QUESTIONS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	C	A	C	B	B	B	A	A	B	C	C	A	C	A	C
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
A	C	C	A	C	C	A	B	D	C	C	D	D	A	C	B
33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
C	A	C	D	A	A	A	C	C	A	B	B	C	B	A	A
49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64
D	B	D	A	A	B	A	D	C	B	D	A	A	C	D	C
65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
C	C	B	B	B	D	A	C	C	C	D	B	A	A	C	B
81	82	83	84	85	86	87	88								
A	D	D	A	B	C	D	A								

SHORT QUESTION'S OF CHAPTER-12
IN ALL PUNJAB BOARDS 2011-2021

Topic I: Trignometric Ratios:

1. Solve the right angle triangle in which a = 3.28, b=5.74, $\gamma = 90°$.

Sol.

a = 3.28

$\gamma = 90°$

By Pathagorous theorem

$(Hyp)^2 = (Base)^2 + (Perp)^2$

$c^2 = b^2 + a^2$

$c^2 = (5.74)^2 + (3.28)^2 = 32.94 + 10.75$

$c^2 = 43.69$

c = 6.6104

$\text{Sin}\alpha = \frac{Par}{Hyp} = \frac{a}{c}$

$\text{Sin}\alpha = \frac{3.28}{6.61} = 0.50,$

$\alpha = \text{Sin}^{-1}(0.50)$

$= 29^\circ\ 45'$

As, $\alpha + \beta + \gamma = 180^\circ$

$29^\circ\ 45' + \beta + 90^\circ = 180^\circ$

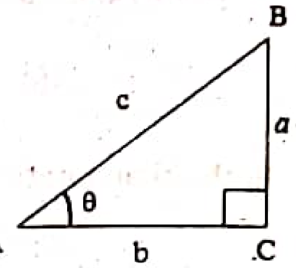
$\beta = 180^\circ - 119^\circ\ 45'$

$\beta = 60^\circ\ 15'$ Ans.

2. In right triangle ABC, find a if b = 30.8, c = 37.2, $\gamma = 90^\circ$

Sol. b = 30.8, c = 37.2, $\gamma = 90^\circ$

From figure



$\text{Cos}\alpha = \frac{b}{c} = \frac{30.8}{37.2}$

$\text{Cos}\alpha = 0.8290$

$\alpha = \text{Cos}^{-1}(0.8290)$

$= 34^\circ\ 7'$

$\because \gamma = 90^\circ$

$\Rightarrow \beta = 90^\circ - \alpha$

$= 90^\circ - 34^\circ\ 7' = 55^\circ\ 54'$

Now $\text{Sin}\alpha = \frac{a}{c}$

$a = c\ \text{Sin}\ \alpha$

$= 37.2\ (\text{Sin}\ 34^\circ 7') = 37.2\ (0.5606) = 20.855 = 20.9$

Hence a = 20.9, $\alpha = 34^\circ\ 7'$, $\beta = 55^\circ 54'$

3. Solve the triangle ABC in which $\gamma=90^\circ$, $\beta = 50^\circ\ 10'$ and c = 0.832.

Sol. $\gamma=90^\circ$, $\beta = 50^\circ\ 10'$ c = 0.832.

a = ?, b = ?

$\Rightarrow \alpha = 90^\circ - \beta = 90^\circ - 50^\circ\ 10' = 39^\circ 50'$

$\alpha = 39^\circ 50'$

$\because \sin\alpha = \frac{a}{c} \Rightarrow a = c \sin a \Rightarrow a = 0.832\ \text{Sin}(39^\circ 50')$

$a = (0.832)(0.64055)$

$a = 0.533$

$\because \cos\alpha = \frac{b}{c}$

$\Rightarrow b = c \cos\alpha = (0.832)(\cos 39^\circ 50') = (0.832)(0.76791)$

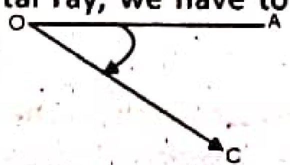
$b = 0.638$

4. Define angle of elevation. (3 times)

Sol. For Looking at B above horizontal ray. We have to raise our eye and m< AOB is called angle of elevation.

5. Define angle of depression (2 times)

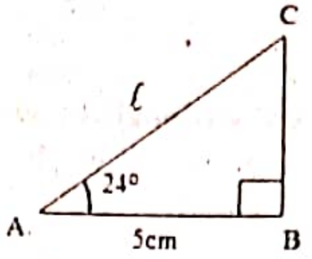
Sol. For Looking at C, below the horizontal ray, we have to Lower our eye and m< AOC is called angle of depression.



6. A ladder learning against a vertical wall makes an angle 24° with the wall. Its foot is 5m form the wall. Find its length. (3 times 2018)

Sol Consider l be the length

Of ladder. Then



$\text{Cos}24^0 = \frac{\overline{AB}}{\overline{AC}}$

$0.9135 = \frac{5}{\ell}$

$\ell = \frac{5}{0.9135}$

$\ell = 5.47$ m

7. In a right angle triangle ABC, a = 5429, c = 6294 and γ = 90°. Find b, α.

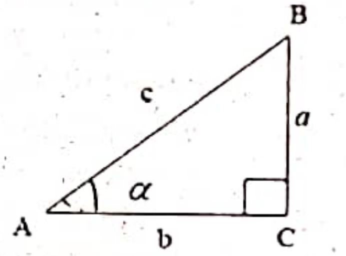
Sol In right angle triangle ABC

Given Find b and α

A = 5429,

C = 6294

$\gamma = 90^0$



Now $Sin\alpha = \frac{a}{c}$

$Sin\alpha = \frac{5429}{6294}$

$Sin\alpha = 0.8626$

$\alpha = Sin^{-1}(0.8626)$

$\alpha = 59^0 \ 36'$

and $\tan\alpha = \frac{a}{b}$

$\tan 59^0 36' = \frac{5429}{b}$

$1.7045 = \frac{5429}{1.7045}$

$b = 3185$

Hence $\alpha = 59^0 36'$, $b = 3185$

8. At the top of a cliff 80 m high, the angle of depression of a boat is 12°. How far is the boat from the cliff. (2 times)

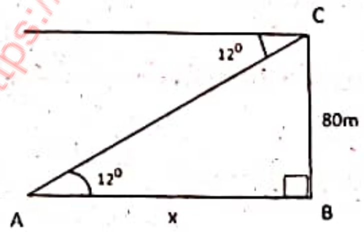
Sol Let x be the required distance

$Tan\theta = \frac{BC}{AB}$

Then $Tan12^0 = \frac{80}{x}$

$0.2126 = \frac{80}{x}$

$x = 376.37m$



Topic II: Solution of Oblique Triangles:

9. Find the measure of the greatest angle, if the sides of the triangle are 16, 20, 33. (4 times)

Sol. a = 16 , b = 20, c = 33

Since c is the greatest side so.

By Law of Cosine

$\cos Y = \frac{a^2+b^2-c^2}{2ab} = \frac{(16)^2+(20)^2-(33)^2}{2(16)(20)} = -0.67656 = \cos^{-1}(-0.67656)$

$Y = 132°34'$

10. Solve the triangle in which a = 32, b = 40, c = 66

Sol. a = 32 , b = 40 , c = 66 $\because 2s = a+b+c$

$S = \frac{a+b+c}{2} = \frac{32+40+66}{2} = \frac{138}{2} = 69$

Now, $\cos\frac{\alpha}{2} = \sqrt{\frac{S(S-a)}{bc}} = \sqrt{\frac{69(69-32)}{(40)(66)}} = 0.98$

$\frac{\alpha}{2} = \cos^{-1}(0.98) = 10.459$

$\Rightarrow \alpha = 2(10.459) = 20.92 = 20°\ 55'$

and

$\cos\frac{\beta}{2} = \sqrt{\frac{S(S-b)}{ac}} = \sqrt{\frac{69(69-40)}{(32)(66)}} = 0.97$

$\frac{\beta}{2} = \cos^{-1}(0.97) = 13.25$

$\Rightarrow \beta = 2(13.25)$

$\beta = 26°\ 30'$

As $\alpha + \beta + \gamma = 180°$

$\gamma = 180° - \alpha - \beta = 180° - 20°\ 55' - 26°\ 30'$

$= 180° - 47°\ 25' = 132°\ 35'$

11. Write the law of Sine. **(5 times)**

Sol. Law of Since

In any ΔABC with usual notation

$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$ is called law of sine.

12. State the Law of Tangents of triangle. **(2 times)**

Sol. The Law of Tangents:

In any triangle ABC with usual notations the law of tangents are given below.

(i) $\frac{a-b}{a+b} = \frac{\tan\frac{\alpha-\beta}{2}}{\tan\frac{\alpha+\beta}{2}}$

(ii) $\frac{b-c}{b+c} = \frac{\tan\frac{\beta-\gamma}{2}}{\tan\frac{\beta+\gamma}{2}}$

(iii) $\frac{c-a}{c+a} = \frac{\tan\frac{\gamma-\alpha}{2}}{\tan\frac{\gamma+\alpha}{2}}$

13. Prove that $\sin\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

Sol. $\sin\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

Proof:

$2\sin^2\frac{\gamma}{2} = 1 - \cos\gamma$

$= 1 - \frac{a^2+b^2-c^2}{2ab}$ $\because \cos\gamma = \frac{a^2+b^2-c^2}{2ab}$

$= \frac{2ab-a^2-b^2+c^2}{2ab} = \frac{c^2-(c^2+b^2-2ab)}{2ab} = \frac{c^2-(a-b)^2}{2ab}$

$= \frac{(c+a-b)(c-a+b)}{2ab} = \frac{(b+c-a)(a+c-b)}{2ab}$

Let $a + b + c = 2s$

$a + b + c - 2a = 2s - 2a$

$b + c - a = 2(s - a)$

So $2\sin^2\frac{\gamma}{2} = \frac{2(s-a)2(s-b)}{2ab}$

$\Rightarrow$ $\sin^2\frac{\gamma}{2} = \frac{2(s-a)2(s-b)}{4ab} = \frac{(s-a)(s-b)}{ab}$

$\Rightarrow$ $\sin\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$ (Hence Proved)

14. Find c if b $= \sqrt{6}$, $\beta = 60°$, $\gamma = 5°$ in triangle ABC. **(6 times)**

Sol. $b = \sqrt{6}$, $\beta = 60°$, $\gamma = 5°$ in triangle ABC

Since $\frac{c}{\sin\gamma} = \frac{b}{\sin\beta}$

$c = \frac{b\sin\gamma}{\sin\beta} = \frac{\sqrt{6}.\sin5^o}{\sin 60^o} = \frac{(2.449489)(0.087156)}{0.86602} = 0.25$

15. State the law of cosine of any two. **(6 times)**

Sol. Law of Cosine be

$a^2 = b^2 + c^2 - 2bc\cos\alpha$ $\longrightarrow$ (i)

$b^2 = a^2 + c^2 - 2ac\cos\beta$ $\longrightarrow$ (ii)

$c^2 = a^2 + b^2 - 2ab\cos\gamma$ $\longrightarrow$ (iii)

(i) $\Longrightarrow$ $\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$

(ii) $\Longrightarrow$ $\cos\beta = \frac{a^2+c^2-b^2}{2ac}$

(ii) $\Longrightarrow$ $\cos\gamma = \frac{a^2+b^2-c^2}{2ab}$

16. Solve the triangle ABC for b = 125, $\gamma = 53°$, and $\alpha = 47°$.

Sol. $b = 125$ $\gamma = 53°$, $\alpha = 47°$

$a = ?$ $c = ?$ $\beta = ?$

$\alpha + \beta + \gamma = 180°$

$47° + \beta + 53° = 180°$ $\Longrightarrow \beta = 180° - 100° = 80^0$

$\frac{a}{\sin a} = \frac{b}{\sin\beta} \Longrightarrow a = \frac{b\sin a}{\sin\beta}$

$a = \frac{(125)\sin47^o}{\sin 80^o} = \frac{(125)(0.731354)}{(0.984808)} = \frac{91.41925}{0.984808} = 92.83$

$\because \frac{c}{\sin\gamma} = \frac{a}{\sin a}$

$c = \frac{a\sin\gamma}{\sin a} = \frac{(125).\sin53^o}{\sin47^o} = \frac{125(0.798635)}{0.731354} = 136.50$

17. Define an oblique triangle.

Sol. A triangle which is not right angled is called an oblique triangle.

18. Find smallest angle of triangle ABC when a= 37.34, b = 3.24, c = 35.06. **(2 times)**

Sol. $a = 37.34$, $b = 3.24$, $c = 35.06$

The smallest angle will be the angle opposite to the smallest side which is β using law of cosine, we have.

$\Longrightarrow$ $\cos\beta = \frac{a^2+c^2-b^2}{2ac}$

$= \frac{(37.34)^2+(35.06)^2-(3.24)^2}{2(37.34)(35.06)}$

$\cos\beta = 0.998$

$\Longrightarrow$ $\beta = \cos^{-1}(0.998)$

$\beta = 3°37'$

19. Solve the right triangle ABC in which $\gamma = 90°$, $\alpha = 37°20'$, a = 243.

(3 times)

Sol. $\alpha = 37°20'$

$\gamma = 90°$

$a = 243$

$\therefore \alpha + \beta = 90°$

$\beta = 90° - \alpha = 90° - 37°20'$

$\beta = 52°40'$

$\because \sin\alpha = \frac{a}{c}$

$\Rightarrow \quad c = \frac{a}{\sin\alpha} = \frac{243}{\sin 37°20'}$

$c = \frac{243}{0.50645} = 400.6$

$c = 400.6$

Also $\cos\alpha = \frac{b}{c}$

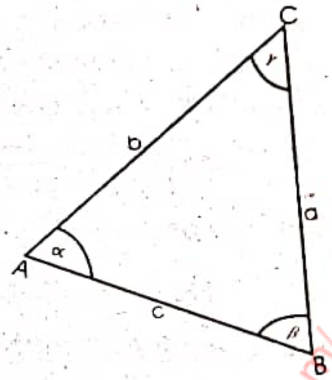
$\Rightarrow \quad b = c\cos\alpha = (400.6)(\cos 37°20')$

$= (400.6)(0.79512)$

$b = 318.5$

20. Prove that $\text{Cos}\frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}}$

Sol. Consider a Triangle ABC



Then

$S = \frac{a+b+c}{2}$

OR $2S = a + b + c$

OR $2S - a = b + c$

OR $2S - b = a + c$

OR $2S - C = a + b$

We know

$\text{Cos } 2\alpha = \text{Cos}^2\alpha - \text{Sin}^2\alpha$

$\text{Cos}\alpha = \text{Cos}^2\frac{\alpha}{2} - \text{Sin}^2\frac{\alpha}{2}$

$\text{Cos}\,\alpha = \text{Cos}^2\frac{\alpha}{2} - (1 - \text{Cos}^2\frac{\alpha}{2})$

$\text{Cos}\alpha = \text{Cos}^2\frac{\alpha}{2} - 1 + \text{Cos}^2\frac{\alpha}{2}$

$\text{Cos}\,\alpha = 2\text{Cos}^2\frac{\alpha}{2} - 1$

$1 + \text{Cos}\alpha = \text{Cos}^2\frac{\alpha}{2}$

OR

$2\text{Cos}^2 s\frac{\alpha}{2} = 1 + \text{Cos}\alpha = 1 + \frac{b^2 + c^2 - a^2}{2bc}$ By Cosine Law

$2\text{Cos}^2 s\frac{\alpha}{2} = \frac{2bc + b^2 + c^2 - a^2}{2bc} = \frac{b^2 + c^2 + 2bc - a^2}{2bc}$

$2\text{Cos}^2 s\frac{\alpha}{2} = \frac{(b+c)^2 - a^2}{2bc} = \frac{(b+c+a)(b+c-a)}{2bc}$

$\text{Cos}^2 s\frac{\alpha}{2} = \frac{(a+b+c)(b+c-a)}{2bc} = \frac{S\,(2S-a-a)}{2bc}$

$\text{Cos}^2 s\frac{\alpha}{2} = \frac{S\,(S-a)2}{bc} = \sqrt{\frac{S\,(S-a)}{bc}}$

21. Solve the Triangle in Which a = 7, b = 3, c = 5. **(2 times)**

Sol. Using Cos Law

$\text{Cos}\alpha = \frac{b^2 + c^2 - a^2}{2bc} = \frac{9+25-49}{2(3)(5)} = \frac{-15}{30}$

$\alpha = \text{Cos}^{-1}\left(\frac{-1}{2}\right) \Rightarrow \alpha = -60° = 180° - 60° = 120°$

$\cos\beta = \frac{a^2+c^2-b^2}{2ac} = \frac{49+25-9}{2(7)(5)} = \frac{65}{70} = \cos^{-1}(0.92) = 21.79^\circ$

For r

$\alpha + \beta + r = 180^\circ$

$120^\circ + 21.79^\circ + r = 180^\circ$

$r = 38.21^\circ$

22. **Solve the Triangle ABC. If b = 125, $\gamma = 53^\circ$, $\alpha = 47^\circ$** **(2 times)**

Sol. $\alpha + \beta + \gamma = 180^\circ$

$53^\circ + \beta + 47^\circ = 180^\circ$

$\beta = 180^\circ - 53^\circ - 47^\circ$

$\beta = 80^\circ$

c = ?

$\frac{c}{\sin\gamma} = \frac{b}{\sin\beta}$

$c = \frac{b}{\sin\beta} \cdot \sin\gamma$

$c = \frac{125}{\sin 80^o} \times \sin 53^o$

$c = \frac{125}{0.985} \times 0.798$

$c = \frac{99.82}{0.985}$

$C = 101.35$

a = ?

$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$

$a = \frac{b}{\sin\beta}\sin\alpha = \frac{125}{\sin 80^o}\sin\alpha = \frac{125}{0.985}\sin 47^o = \frac{125}{0.985} \times 0.73 = \frac{91.42}{0.985} = 92.81$

23. **Find the smallest angle of Δ ABC when a = 37.34 , b = 3.24 , c = 35.06** **(5 times)**

Sol. We know Smallest angle is b/w two Largest sides which are a & c

$\therefore \beta$ is the smallest angle

$\cos\beta = \frac{a^2+c^2-b^2}{2ac}$

$\cos\beta = \frac{(37.34)^2+(35.06)^2-(3.24)^2}{2(37.34)(35.06)}$

$\cos\beta = \frac{2612.9816}{2618.2808}$

$\beta = \cos^{-1}(0.9979)$

$\beta = 3.65^\circ$

24. **Solve the ΔABC in which a = 3, c = 6 and $\beta = 36^0 20'$.**

Sol ΔABC in which

Given	Find
a=3	b=?
c=6	α=?
$\beta = 36^0\ 20'$	γ=?

Using law of Cosine

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$b^2 = (3)^2 + (6)^2 - 2(3)(6)\cos 36^\circ 20'$

$b^2 = 9 + 36 - 36(0.8056)$

$b^2 = 45 - 29.001$

$b^2 = 15.99$

$b = 3.99 \quad \Rightarrow b = 4$

Using law of sine.

$$\frac{a}{\sin\alpha}=\frac{b}{Sin\beta}$$

$$\frac{3}{Sin\alpha}=\frac{4}{Sin36^0 20'}$$

$$\frac{3}{Sin\alpha}=\frac{4}{0.5924} \quad \Rightarrow \quad 3=6.751Sin\alpha$$

$$Sin\alpha=\frac{3}{6.751}$$

$$Sin\alpha=0.4443$$

$$\alpha=Sin^{-1}(0.4443)$$

$$\alpha=26^0 22'$$

We know that

$$\alpha+\beta+\gamma=180^0$$

$$26^0 22'+36^0 20'+\gamma=180^0$$

$$62^0 42'+\gamma=180^0$$

$$\gamma=180^0-62^0 42'=117^0 18'$$

25. By usig the law of sines show that $\Delta=\dfrac{c^2\sin\alpha\sin\beta}{2\sin\gamma}$

Sol $\quad \Delta=\dfrac{c^2\sin\alpha\sin\beta}{2\sin\gamma}$

By the law of sines, we know that

$$\frac{a}{Sin\,\alpha}=\frac{b}{Sin\,\beta}=\frac{c}{Sin\,\gamma}$$

$$\Rightarrow \quad a=c\frac{Sin\,\alpha}{Sin\,\gamma} \quad and \quad b=c\frac{Sin\,\beta}{Sin\,\gamma}$$

We know that area of triangle, ABC is

$$\Delta=\frac{1}{2}abSin\gamma$$

$$\Delta=\frac{1}{2}\left(c\frac{Sin\alpha}{Sin\gamma}\right)\left(c\frac{Sin\beta}{Sin\gamma}\right)Sin\gamma$$

$$=\frac{c^2Sin\alpha\,Sin\beta}{2\,Sin\gamma}$$

$$=\text{R.H.S}$$

$$\text{L.H.S}=\text{R.H.S}$$

Topic III: Area of Triangle:

26. Find the Area of the triangle if a = 200 ; b = 120 , Y = 150°. **(9 times)**

Sol. a = 200 , b = 120 , Y = 150°

We know that

Area of Δ ABC = $\frac{1}{2}$ ab Sin Y

= $\frac{1}{2}$ (200) (120) Sin 150°= 120000 (0.5)= 6000 sq. units

27. Find the area of the triangle ABC when b = 21.6, c = 30.2 and $\alpha = 50°40'$.

Sol. b = 21.6 , c = 30.2 and $\alpha = 50°40'$

Area of triangle ABC $= \frac{1}{2}$ bc Sin α

$= \frac{1}{2}(21.6)(30.2) \sin 50°40'$

= 252.3 sq. unit

28. If in a Δ ABC, a = 93; c = 101; $\beta = 80°$ find b

Sol. Let a = 93 , c = 101 , $\beta = 80°$

We know that

$b^2 = c^2 + a^2 - 2ca\cos\beta$

$b^2 = (101)^2 + (93)^2 - 2(101)(93)\cos 80°$

$b^2 = 10201 + 8649 - 18786\,(0.1736)$

$b^2 = 18850 - 3262.15 = 15587.85$

Taking square root b = 124.85 = 125

29. Find the area of a triangle ABC, given two sides and internal angle,

a = 4.33 , b = 9.25, $\gamma = 56°44'$

Sol. a = 4.33 , b = 9.25, $\gamma = 56°44'$

Here two sides and one angle are given so the required area is

$\Delta = \frac{1}{2}ab\sin\gamma$

$= \frac{1}{2}(4.33)(9.25)(\sin 56°44')$

$= \frac{1}{2}(4.33)(9.25)(0.836) = 16.74$

Δ = 16.74 sq units.

30. Find the area of the triangle ABC , when a = 18, b = 24, c = 30. (6 times)

Sol. a = 18 b = 24 c = 30

$s = \frac{a+b+c}{2} = \frac{18+24+30}{2} = \frac{72}{2} = 36$

$s - a = 36 - 18 = 18$

$s - b = 36 - 24 = 12$

$s - c = 36 - 30 = 6$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta = \sqrt{(36)(18)(12)(6)} = \sqrt{46656} = 216$ sq. units.

31. Find the area of ΔABC for b = 25.4, $\gamma = 36°41'$ and a = 45°17'. (2 times)

Sol. b = 25.4, $\gamma = 36°41'$ a = 45°17'

$\therefore \alpha + \beta + \gamma = 180°$

$\Rightarrow 45°17' + \beta + 36°41' = 180° \Rightarrow \beta = 180^0 - 81^0 58' = 98^0 2'$

$\Delta = \frac{1}{2}\frac{b^2 \sin\alpha \sin\gamma}{\sin\beta} = \frac{1}{2}\frac{(25.4)^2 \sin 45°17' \sin 36°41}{\sin 98°2'}$

$\Delta = \frac{1}{2}\frac{(645.16)(0.72059)(9.59739)}{0.99018} = 138.29$ Sq. unit

32. The Area of Triangle is 2437.IF a = 79 , c = 97 , Find angle β. (6 times)

Sol. We know

$\frac{1}{2}ac\sin\beta$ = Area of Triangle

$\frac{1}{2}(79)(97)\sin\beta = 2437$

$\frac{7663}{2}\sin\beta = 2437$

$3831.5\sin\beta = 2437$

$\sin\beta = \frac{2437}{3831.5}$

$\beta = \sin^{-1}(0.636) \Rightarrow \beta = 39.50°$

33. **If Area Triangle is 121.34** $\alpha = 32°15'$, $\beta = 65°37'$ Find C **(2 times)**

Sol. $\alpha = 32°15' = 32.25°$ $\beta = 65°37' = 65.62°$

$\alpha + \beta + \gamma = 180°$

$32.25° + 65.62° + \gamma = 180°$

$\gamma = 82.13°$

$32.25° + 65.62° + \gamma = 180°$

$\gamma = 82.13°$

Area of Triangle $= \frac{1}{2} c^2 \frac{Sin\alpha Sin\beta}{Sin\gamma}$

$121.34 = \frac{1}{2} c^2 \frac{Sin32.25° \times Sin65.62°}{Sin82.13°}$

$242\text{-}68 = c^2 \left(\frac{0.4823}{0.99}\right)$

$242.68 = c^2 (0.487)$

$\frac{242.68}{0.418} = c^2$

$498.14 = c^2 \Rightarrow c = 22.32$

34. **Find area of triangle ABC when a = 4.8, α = 83°42′, γ = 37°12′.**

Sol Given $a = 4.8$, $\alpha = 83°42'$, $\gamma = 37°12'$.

We know that

$$\alpha + \beta + \gamma = 180^0$$

$$\beta = 180^0 - \alpha - \gamma$$

$$\beta = 180^0 - 83^0 42' - 37^0 12'$$

$$\beta = 59^0 6'$$

We know that area of triangle

$$\Delta = \frac{a^2 Sin\beta\, Sin\gamma}{2Sin\alpha}$$

$$\Delta = \frac{(4.8)^2\ Sin59^0\, 6'\ Sin37^0\, 12'}{2Sin83^0\, 42'}$$

$$\Delta = \frac{(23.04)\ (0.858)(0.605)}{2(0.994)}$$

$$\Delta = \frac{11.9598}{1.988}$$

$\Delta = 6.02$ Square units

35. **Find the area of ΔABC, if b = 37, c=45 and α = 30°50′.** **(5 times)**

Sol Given

$b = 37,\ c = 45,\ \alpha = 30^0\, 50'$

We know that area of triangle ABC is

$$\Delta = \frac{1}{2} bc\ Sin\,\alpha$$

$$= \frac{1}{2}(37)(45)\,Sin\,30^0\,50'$$

$= (832.5)(0.5125)$

$\Delta = 426.69$ Square units

36. **Find the area of triangle ABC if a = 524 b = 276 c= 315.** **(2 times)**

Sol Given

A = 524, b = 276, c = 315

We know that

$$S=\frac{a+b+c}{2}=\frac{524+276+315}{2}=\frac{1115}{2}=557.5$$

$$S-a=557.5-524=33.5$$

$$S-b=557.5-276=281.5$$

$$S-c=557.5+315=242.5$$

Now $\Delta=\sqrt{S(S-a)(S-b)(S-c)}$

$$=\sqrt{(557.5)(33.5)(281.5)(242.5)}$$

$$=\sqrt{1274910861}$$

$\Delta=35705.9$ square units

Topic IV: Circles Connected with Triangle:

37. Sides of a triangle ABC are a = 13 , b = 14 , c = 15. Find R and r_1. **(5 times)**

Sol. a = 13, b = 14, c = 15

$$S=\frac{a+b+c}{2}=\frac{13+14+15}{2}$$

$$S=\frac{42}{2}=21$$

$$\Delta=\sqrt{S(S-a)(S-b)(S-c)}$$

$$\Delta=\sqrt{21(8)(7)(6)}=\sqrt{7056}=84$$

$$R=\frac{abc}{4\Delta}=\frac{(13)(14)(15)}{4(84)}=\frac{65}{8}$$

$$r_1=\frac{\Delta}{S-a}=\frac{84}{8}=\frac{21}{2}$$

38. Define Circum-circle. **(2 times)**

Sol. Circum – Circle :

The circle passing through three vertices of a triangle is called circum circle. Its centre is called circum centre. And its radius is called circum radius.

39. Show that $r=(s-\alpha)\tan\frac{\alpha}{2}$ **(4 times)**

Sol. $r=(s-\alpha)\tan\frac{\alpha}{2}$

Proof: We know that

$$\tan\frac{\alpha}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

Now.

R.H.S $=(s-a)\tan\frac{\alpha}{2}=(s-a)\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}=\sqrt{\frac{(S-a)^2(S-b)(S-c)}{S(S-a)}}$

$$=\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}=\sqrt{\frac{s(s-a)(s-b)(s-c)}{s^2}}=\frac{\Delta}{s}=r=\text{L.H.S}$$

40. Prove that $\gamma_1\gamma_2\gamma_3=\gamma s^2$. **(3 times)**

Sol. $r_1 r_2 r_3 = rs^2$

Proof: L.H.S $=r_1r_2r_3=\left(\frac{\Delta}{s-a}\right)\left(\frac{\Delta}{s-b}\right)\left(\frac{\Delta}{s-c}\right)$

$=\frac{\Delta^3}{(s-a)(s-b)(s-c)}$ $\Longrightarrow$ $=\frac{s\Delta^3}{s(s-a)(s-b)(s-c)}$

$=\frac{s\Delta^3}{\Delta^2}$ $(\because \Delta^2=s(s-a)(s-b)(s-c))$

$=s\Delta$ $=s^2\frac{\Delta}{s}$ $=s^2r$

$r_1r_2r_3 = rs^2$ Proved

41. In triangle ABC, a = 34, b = 20 , c = 42. Find R and r.

Sol. $a = 34, b = 20, c = 42$

$$s = \frac{a+b+c}{2} = \frac{34+20+42}{2} = \frac{96}{2} = 48$$

$$s - a = 48 - 34 = 14$$

$$s - b = 48 - 20 = 28$$

$$s - c = 48 - 42 = 6$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{48.14.28.6} = 336$$

$$R = \frac{abc}{4\Delta} = \frac{(34)\ (20)\ (42)}{4(336)} = \frac{85}{4} = 21.25$$

$$r = \frac{\Delta}{s} = \frac{336}{48} = 7$$

42. Show that $r_2 = S\ Tan\frac{\beta}{2}$

Sol. R.H.S $= s\ Tan\frac{\beta}{2}$

$$= s\sqrt{\frac{(S-a)(S-c)}{S(S-b)}} = s\sqrt{\frac{S(S-a)(S-b)(S-c)}{S(S-b)S(S-b)}} = s\sqrt{\frac{S(S-a)(S-b)(S-c)}{S^2(S-b)^2}}$$

$$= \frac{S\sqrt{S(S-a)(S-b)(S-c)}}{S(S-b)} = \frac{\Delta}{S-b} = r_2 = \text{L.H.S}$$

43. Show that $r = (S-b)\ Tan\frac{\beta}{2}$

Sol. R.H.S $= (S-b)\ Tan\frac{\beta}{2}$

$$= (S-b)\sqrt{\frac{(S-a)(S-c)}{S(S-b)}} = (S-b)\sqrt{\frac{S(S-a)(S-b)(S-c)}{S(S-b)S(S-b)}}$$

$$= (S-b)\frac{\sqrt{S(S-a)(S-b)(S-c)}}{S(S-b)}$$

$$= \frac{\sqrt{S(S-a)(S-b)(S-c)}}{S} = \frac{\Delta}{S} = r = \text{L.H.S}$$

44. Prove that $R = \frac{abc}{4\Delta}$ **(4 times)**

Sol. We Know $R = \frac{a}{2\ Sin\alpha}$

$$R = \frac{a}{2\left(2Sin\frac{\alpha}{2}Cos\frac{\alpha}{2}\right)} = \frac{a}{4\sqrt{\frac{(S-b)(S-c)}{bc}}\sqrt{\frac{S(S-a)}{bc}}} = \frac{a}{\sqrt{\frac{S(s-a)(s-b)(s-c)}{b^2c^2}}}$$

$$R = \frac{a}{\frac{4\sqrt{S(S-a)(s-b)(s-c)}}{bc}} = \frac{abc}{4\sqrt{S(s-a)(s-b)(s-c)}} = \frac{abc}{4\Delta} \because \sqrt{S(s-a)(s-b)(s-c)} = \Delta$$

Proved

45. Find R , if a = 13 , b = 14 , c = 15 **(3 times)**

Sol. $S = \frac{a+b+c}{2}$

$$S = \frac{13+14+15}{2} \Rightarrow S = \frac{42}{2} = 21$$

$$\therefore \Delta = \sqrt{S(S-a)(S-b)(S-c)}$$

$$\Delta = \sqrt{21(21-13)(21-14)(21-15)} = \sqrt{21 \times 8 \times 7 \times 6} = \sqrt{7056} = 84$$

Hence

$$R = \frac{abc}{4\Delta} = \frac{(13)(14)(15)}{4 \times 84} = \frac{2730}{336.00} = 8.125$$

46. Define In Circle. **(2 times)**

Sol. A Circe which touches the sides of Triangle internally is called In Circle.

47. It measures of sides of ΔABC are 17,20,21, Find r.

Sol. a = 17 , b = 20 c = 21

and $S = \frac{a+b+c}{2} = \frac{17+20+21}{2} = 29$

$S = 24$

$\Delta = \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{29(29-17)(29-20)(29-21)}$

$= \sqrt{29(12)(9)(8)} = \sqrt{25056} = 158.29$ Ans.

48. Prove that $r = b \text{ Sin}\frac{\gamma}{2} \text{ Sin}\frac{\alpha}{2} \text{ Sec}\frac{\beta}{2}$.

Sol. R.H.S

$b \text{ Sin}\frac{\gamma}{2} \text{ Sin}\frac{\alpha}{2} \text{ Sec}\frac{\beta}{2}$.

$= b\sqrt{\frac{(S-a)(S-b)}{ab}} \cdot \sqrt{\frac{(S-b)(S-c)}{bc}} \cdot \sqrt{\frac{ac}{S(S-b)}}$

$= b\sqrt{\frac{(S-a)(S-b)^2(S-c)(ac)}{(ab^2c)S(S-b)}} = \frac{b}{b}\sqrt{\frac{(S-a)(S-b)(S-c)}{S}}$

$= \sqrt{\frac{S(S-a)(S-b)(S-c)}{S.S}}$

$= \frac{\sqrt{S(S-a)(S-b)(S-c)}}{S}$ $\because \Delta = \sqrt{S(S-a)(S-b)(S-c)}$

$= \frac{\Delta}{S} = r =$ L.H.S $\because r = \frac{\Delta}{S}$

49. Prove $r_3 = S\tan\frac{\gamma}{2}$

Sol $r_3 = S\tan\frac{\gamma}{2}$

$\text{R.H.S} = S\tan\frac{\gamma}{2}$

$= S\sqrt{\frac{(S-a)(S-b)}{S(S-c)}}$

$= S\sqrt{\frac{S(S-a)(S-b)(S-c)}{S^2(S-c)^2}}$

$= S\frac{\sqrt{S(S-a)(S-b)(S-c)}}{S.(S-c)}$

$= \frac{\Delta}{S-c}$

$= r_3$

= L.H.S

L.H.S = R.H.S

50. Prove that $(r_3 - r)\,Cot\frac{\gamma}{2} = C$

Sol $(r_3 - r)\,Cot\frac{\gamma}{2} = C$

$L.H.S = (r_3 - r)\,Cot\frac{\gamma}{2}$

$= \left(\frac{\Delta}{S-c} - \frac{\Delta}{S}\right)\sqrt{\frac{S(S-c)}{(S-a)(S-b)}}$

$$= \Delta\left(\frac{1}{S-c}-\frac{1}{S}\right)\sqrt{\frac{S^2(S-c)^2}{S(S-a)(S-b)(S-c)}}$$

$$= \Delta\left(\frac{1}{S-c}-\frac{1}{S}\right)\frac{S(S-c)}{\sqrt{S(S-a)(S-b)(S-c)}}$$

$$= \Delta\left(\frac{S-(S-c)}{S(S-c)}\right)\frac{S(S-c)}{\Delta}$$

$$= S-S+c$$

$$= c$$

L.H.S = R.H.S

51. Show that $r = a\,Sin\frac{\beta}{2}\;Sin\frac{\gamma}{2}\;Sec\frac{\alpha}{2}$

Sol $r = aSin\frac{\beta}{2}\;Sin\frac{\gamma}{2}\;Sec\frac{\alpha}{2}$

$$\text{R.H.S} = aSin\frac{\beta}{2}\;Sin\frac{\gamma}{2}\;Sec\frac{\alpha}{2}$$

$$= a\sqrt{\frac{(s-a)(S-c)}{ac}}\sqrt{\frac{(s-a)(S-b)}{ab}}\sqrt{\frac{bc}{S(S-a)}}$$

$$= a\sqrt{\frac{(S-a)(S-b)(S-c)}{a^2S}}$$

$$= a\sqrt{\frac{S(S-a)(S-b)(S-c)}{a^2S^2}}$$

$$= a\frac{\sqrt{S(S-a)(S-b)(S-c)}}{aS}$$

$$\because \sqrt{S(S-a)(S-b)(S-c)} = \Delta$$

$$= \frac{\Delta}{S} = r = L.H.S$$

$L.H.S = R.H.S$

2018

52. With Usal notation show that: $\frac{1}{r} = \frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$ **(2 times)**

Sol: R.H.S

$$= \frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$$

$$= \frac{1}{\frac{\Delta}{s-a}}+\frac{1}{\frac{\Delta}{s-b}}+\frac{1}{\frac{\Delta}{s-c}}$$

$$= \frac{s-a}{\Delta}+\frac{s-b}{\Delta}+\frac{s-c}{\Delta}$$

$$= \frac{s-a+s-b+s-c}{\Delta}$$

$$= \frac{3s - (a+b+c)}{\Delta}$$

$$= \frac{3s - 2s}{\Delta}$$

$$= \frac{s}{\Delta} = \frac{1}{\Delta/s} = \frac{1}{r} = L.H.S$$

53. **Prove that** $(r_1 + r_2)\tan\frac{\alpha}{2} = c$ **(2 times)**

Sol: L.H.S

$$(r_1 + r_2)\tan\frac{\gamma}{2}$$

$$= \left(\frac{\Delta}{s-a} + \frac{\Delta}{s-b}\right)\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

$$= \frac{\Delta(s-b) + \Delta(s-a)}{(s-a)(s-b)}\sqrt{\frac{(s-a)(s-b)}{s(s-c)} \cdot \frac{s(s-c)}{s(s-c)}}$$

$$= \Delta\left[\frac{s-b+s-a}{(s-a)(s-b)}\right]\sqrt{\frac{s(s-a)(s-b)(s-c)}{s^2(s-c)^2}}$$

$$= \Delta\left[\frac{2s-b-a}{(s-a)(s-b)}\right]\sqrt{\frac{\Delta^2}{s^2(s-c)^2}}$$

$$= \Delta\left(\frac{2s-b-a}{(s-a)(s-b)}\right)\left(\frac{\Delta}{s(s-c)}\right)$$

$$= \Delta^2\left(\frac{a+b+c-a-b}{s(s-a)(s-b)(s-c)}\right)$$

$$= \frac{\Delta^2 c}{s(s-a)(s-b)(s-c)}$$

$$= \frac{\Delta^2 c}{\Delta^2} \quad c = R.H.S$$

54. **Show that** $r_1 = s\tan\frac{\alpha}{2}$

Sol: R.H.S

$$= s\tan\frac{\alpha}{2}$$

$$= s\sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = s\sqrt{\frac{(s-b)(s-c)}{s(s-a)} \cdot \frac{s(s-a)}{s(s-a)}} = s\sqrt{\frac{(s-b)(s-c)(s-a)}{s^2(s-a)^2}}$$

$$= s\sqrt{\frac{\Delta^2}{s^2(s-a)^2}} = \frac{s\Delta}{s(s-a)} = \frac{\Delta}{s-a} = r_1 = \text{L.H.S}$$

2019

55: **In the triangle ABC, if** $\alpha = 35^0 17', \beta = 45^0 13'$ **and** $b = 421$. **Find a.**

Sol: Given $\alpha = 35^0 17' = \beta = 45^0 13'$, $b = 421$

By law of Sines

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}$$

$$\frac{a}{Sin35^0 17'}=\frac{b}{Sin45^0 13'}$$

$$\frac{a}{0.5776}=\frac{421}{0.7098}$$

$$a=\frac{421(0.5776)}{0.7098}$$

$$a=342.58$$

Approximately $a=343$

56: When the angle between the ground and the sum is 30^0, flag pole casts a shadow of 40 m long. Find the height of the top of the flag. (2 times)

Sol: Let h be the height of the top of flag.

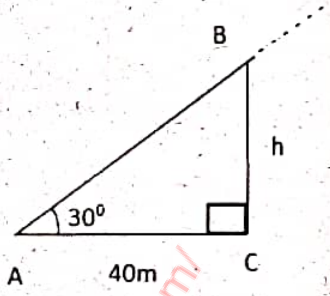
So $\tan\theta=\frac{\overline{BC}}{\overline{AC}}$

$$\Rightarrow \tan 30^0=\frac{h}{40}$$

$$\Rightarrow h=40(\tan 30^0)$$

$$\Rightarrow h=40(0.577)$$

$$\Rightarrow h=23.1\ m$$



57: In any triangle ΔABC, if $C=16.1$, $\alpha=42^0 45'$, $r=74^0 32'$, then find 'β' and 'α'

Sol: Given $C=16.1$, $\alpha=42^0 45'$, $r=74^0 32'$

We know that

$$\alpha+\beta+r=180^0$$

$$42^0 45'+\beta+74^0 32'=180^0$$

$$117^0 17'+\beta=180^0$$

$$\beta=180^0-117^0 17'$$

$$\beta=62^0 43'$$

By law of Sines

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\gamma}$$

$$\frac{\alpha}{Sin42^0 45'}=\frac{16.1}{Sin74^0 32'}$$

$$\frac{\alpha}{0.6788}=\frac{16.1}{0.9637}$$

$$a=\frac{16.1(0.6788)}{0.9637}$$

$$\Rightarrow a=\frac{10.92}{0.9637}$$

$$\Rightarrow a=11.34$$

58: Show that $r = 4R\ Sin\frac{\alpha}{2} Sin\frac{\beta}{2} Sin\frac{\gamma}{2}$ with usual notation.

Sol: $r = 4R\ Sin\frac{\alpha}{2} Sin\frac{\beta}{2} Sin\frac{\gamma}{2}$

R.H.S = $4R Sin\frac{\alpha}{2} Sin\frac{\beta}{2} Sin\frac{\gamma}{2}$

$= \not{4}\left(\frac{abc}{\not{4}\Delta}\right)\sqrt{\frac{(S-b)(S-c)}{bc}}\sqrt{\frac{(S-a)(S-c)}{ac}}\sqrt{\frac{(S-a)(S-b)}{ab}}$

$= \frac{\not{abc}}{\Delta}\frac{(S-a)(S-b)(S-c)}{\not{abc}}$

$= \frac{(S-a)(S-b)(S-c)}{\Delta}$

Multiplying and dividing by S

$= \frac{S(S-a)(S-b)(S-c)}{S\Delta}$

$= \frac{\Delta^2}{S\Delta}$ $\qquad \because \Delta = \sqrt{S(S-a)(S-b)(S-c)}$

$= \frac{\Delta}{S}$ $\qquad \because r = \frac{\Delta}{S}$

$= r$

= L.H.S

So L.H.S = R.H.S

59: Prove that $\Delta = \sqrt{S(S-a)(S-b)(S-c)}$

Sol: We know that

$\Delta = \frac{1}{2}$ bc $Sin\alpha$

$\because Since\ \ Sin\alpha = 2Sin\frac{\alpha}{2}Cos\frac{\alpha}{2}$

So $\Delta = \frac{1}{\not{2}} bc\ 2Sin\alpha/_2 Cos\alpha/_2$

$= bc\ Sin\alpha/_2 Cos\alpha/_2$

$= bc\sqrt{\frac{(S-b)(S-c)}{bc}}\sqrt{\frac{S(S-a)}{bc}}$

$= bc\sqrt{\frac{S(S-a)(S-b)(S-c)}{b^2c^2}}$

$= \not{bc}\frac{\sqrt{S(S-a)(S-b)(S-c)}}{\not{bc}}$

$\Delta = \sqrt{S(S-a)(S-b)(S-c)}$

60: If $\beta = 52^0$, $r = 89^0 35'$, $a = 89.35$ find the side of a ΔABC

Sol: Given $\beta = 52^0$, $r = 89^0 35'$, $a = 89.35$

We know that

$\alpha + \beta + r = 180^0$

$\alpha + 52^0 + 89^0 35' = 180^0$

$\alpha + 141^0 35' = 180^0$

$\alpha = 180^0 - 141^0 35'$

$\alpha = 38^0 25'$

By law of Sines $\dfrac{a}{Sin\alpha} = \dfrac{b}{Sin\beta}$

$\dfrac{89.35}{Sin 38^0 25'} = \dfrac{b}{Sin 52^0}$

$\dfrac{89.35}{0.6214} = \dfrac{b}{0.7880}$

$143.788 = \dfrac{b}{0.7880}$

$b = 143.788(0.7880)$

$b = 113.31$

61: In the right angled triangle ABC if $r = 90^0$, $\alpha = 58^0 13'$, $b = 125.7$. **Find a**

Sol: Given $r = 90^0$, $\alpha = 58^0 13'$, $b = 125.7$

We know that

$\dfrac{a}{b} = \tan \alpha$

$a = b \tan \alpha$

$a = 125.7(\tan 58^0 13')$

$a = 125.7(1.6139) = 202.865$

$\Rightarrow a = 202.9$

62: Define escribed circle.

Sol: **Definition:**

A circle, which touches one side of the triangle externally and the other two produced sides, is called escribed circle.

2021

63- Find θ if sin θ = 0.5791

Sol: $\sin \theta = 0.5791$

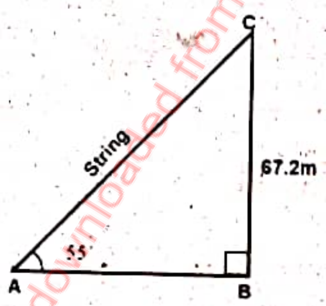
$\theta = \sin^{-1}(0.5791)$

$\theta = 35''23'$

64- A kite is flying at a height of 67.2m is attached to a fully stretched string inclined at an angle of 55° to the horizontal. Find the length of string.

Sol Let the length of string = AC = lm , $\alpha = 55^0$

In right $\triangle ABC$



$\sin \alpha = \dfrac{BC}{AC}$

$$\sin 55" = \frac{67.2}{l}$$

$$0.8192 = \frac{67.2}{l}$$

$$l = \frac{67.2}{0.8192} \quad , \quad l = 82.03m$$

65- **Solve for C in a triangle $\triangle ABC$ if $\gamma = 90^{o}, \alpha = 62^{o}40' and\, b = 796$**

Sol: Given $r = 90"$, $\alpha = 60"40'$, $b = 796$

We know that

$$\cos\alpha = \frac{b}{c}$$

$$\cos 62"40' = \frac{796}{c}$$

$$c = \frac{796}{\cos 62"40'} = \frac{796}{0.4592} = 1733$$

66- **In an equilateral triangle find the value of R**

Sol: Since triangle is an equilateral triangle. $\because a = b = c$

So, $S = \frac{a+a+a}{2} = \frac{3a}{2}$

$$S - a = \frac{3a}{2} - a = \frac{a}{2}$$

Similarly $S - b = \frac{a}{2}$ and $S - c = \frac{a}{2}$

Now $\triangle = \sqrt{S(S-a)(S-b)(S-c)}$

$$= \sqrt{\frac{3a}{2}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)} = \sqrt{\frac{3a^4}{16}}$$

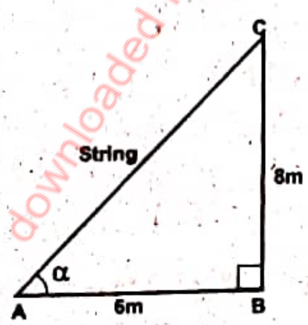
$$\triangle = \frac{\sqrt{3}a^2}{4}$$

$$R = \frac{abc}{4\triangle} = \frac{a.a.a}{4\frac{\sqrt{3}a^2}{4}} = \frac{a}{\sqrt{3}}$$

67- **A vertical pole is 8m high and length of its shadow is 6m .What is the angle of elevation of the sun at that moment?**

Sol: Let required angle of elevation $= \alpha$

In right $\triangle ABC$



$$\tan\alpha = \frac{BC}{AB}$$

$$\tan\alpha = \frac{8}{6}$$

$$\alpha = \tan^{-1}\left(\frac{8}{6}\right)$$

$$\alpha = \tan^{-1}(1.333)$$

$$\alpha = 53^{\circ}8'$$

68- **Prove that** $\frac{1}{r^2}+\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}=\frac{a^2+b^2+c^2}{\Delta^2}$

Sol: L.H.S $=\frac{1}{r^2}+\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}$

$$=\frac{1}{\left(\frac{\Delta}{S}\right)^2}+\frac{1}{\left(\frac{\Delta}{S-a}\right)^2}+\frac{1}{\left(\frac{\Delta}{S-b}\right)^2}+\frac{1}{\left(\frac{\Delta}{S-c}\right)^2}$$

$$=\frac{S^2}{\Delta^2}+\frac{(S-a)^2}{\Delta^2}+\frac{(S-b)^2}{\Delta^2}+\frac{(S-c)^2}{\Delta^2}$$

$$=\frac{S^2+(S-a)^2+(S-b)^2+(S-c)^2}{\Delta^2}$$

$$=\frac{S^2+S^2+a^2-2as+S^2+b^2-2bs+S^2+c^2-2cs}{\Delta^2}$$

$$=\frac{4S^2-2s(a+b+c)+a^2+b^2+c^2}{\Delta^2}$$

$$=\frac{4S^2-2s(2s)+a^2+b^2+c^2}{\Delta^2} \quad \because 2s=a+b+c$$

$$=\frac{\cancel{4S^2}-\cancel{4S^2}+a^2+b^2+c^2}{\Delta^2}=\frac{a^2+b^2+c^2}{\Delta^2}$$

= R.H.S

Hence L.H.S = R.H.S

69- **Find the area of the triangle ABC given three sides: a = 32.65 , b = 42.81 , c = 64.92**

Sol: a = 32.62 , b = 42.81 , C = 64.92

We know that

$$S=\frac{a+b+c}{2}=\frac{32.62+42.81+64.92}{2}=\frac{140.38}{2}=70.19$$

Now,

$$S-a=70.19-32.65=37.54$$

$$S-b=70.19-42.81=27.38$$

$$S-c=70.19-64.92=5.27$$

Area of triangle ABC

$$\Delta=\sqrt{S(S-a)(S-b)(S-c)}$$

$$=\sqrt{7019(37.54)(27.38)(5.27)}=\sqrt{380201.27}=616.60\,sq.unit$$

70- **Find the value of r if a = 34, b = 20 and c = 42** **(2 Times)**

Sol: Given a = 34, b = 20, c = 42

We know that

$$S=\frac{a+b+c}{2}=\frac{34+20+42}{2}=\frac{96}{2}=48$$

$$S-a=48-34=14$$
$$S-b=48-20=28$$
$$S-c=48-42=6$$

Now, $\Delta=\sqrt{S(S-a)(S-b)(S-c)}$

$$\Delta=\sqrt{48(14)(28)(6)}$$
$$\Delta=\sqrt{112896}$$
$$\Delta=336$$

We know that

$$r=\frac{\Delta}{S}=\frac{336}{48}=7$$

71- Find the value of Cot 89° 9′.

Sol: $\cot 89^{o}9'$

$$=\frac{1}{\tan 89^{o}9'}$$
$$=\frac{1}{67.401}=0.0148$$

72- Prove that abc (sin α + sin β + sin γ) $4\Delta S$

Sol: L.H.S $=abc(\sin\alpha+\sin\beta+\sin\gamma)$ $\quad\because \Delta=\frac{1}{2}bc\sin\alpha$

$$=abc\left[\frac{2\Delta}{bc}+\frac{2\Delta}{ac}+\frac{2\Delta}{ab}\right] \quad \because \sin\alpha=\frac{2\Delta}{bc}$$

$$=abc(2\Delta)\left[\frac{1}{bc}+\frac{1}{ac}+\frac{1}{ab}\right] \quad =(2\Delta)abc\left[\frac{a+b+c}{abc}\right]$$

$$=(2\Delta)(a+b+c)=(2\Delta)(2S) \quad \because 2S=a+b+c$$

$=4\Delta S$ = R.H.S

Hence L.H.S = R.H.S

73- Find the value of $\sin 53^{o}40'$

$Sin53^{o}40'$

= $Sin(53.667)$ $=0.8058$

Hence $Sin53^{o}40'$ = 0.8058

74- Find the value of α if a = 7 ; b = 7, c = 9

Sol: Given

a = 7 , b = 7 , c =49

We know that

$$\cos\alpha=\frac{b^2+c^2-a^2}{2bc}=\frac{(7)^2+(9)^2-(7)^2}{2(7)(9)}=\frac{49+81-49}{126}=\frac{81}{126}$$

$$\cos\alpha=0.6429$$
$$\alpha=\cos^{-1}(0.6429)$$
$$\alpha=50^{o}$$

LONG QUESTION'S OF CHAPTER-12 IN ALL PUNJAB BOARDS 2011-2021

Topic I: Trignometric Ratios:

1. A kite flying at a height of 67.2m is attached to a fully stretched string inclined at an angle of 55° to the horizontal. Find the length of the sting.
2. Three villages A,B and C are connected by straight roads 6km. 9km and 13km. What angles these roads make with each other?

Topic II: Solution of Oblique Triangles:

3. Solve the triangle ABC, if c = 16.1 , $\alpha = 42° 45'$, $\gamma = 74°32'$
4. Solve the triangle ABC if a = 53 , $\beta = 88°36'$ $\gamma = 31°54'$ **(2 times)**
5. State and prove the Law of Cosine.
6. The sides of a triangles are $x^2 + x + 1$, $2x + 1$ and $x^2 - 1$. Prove that the greatest angle of the triangle is 120°. **(4 times)**
7. State and prove the law of Sins. (3 times)

Topic III: Area of Triangle:

8. Solve the ΔABC in which b = 14.8, c = 16.1 and $\alpha = 42° 45'$.
9. Find the area of triangle ABC, given three sides: a= 32.65 , b = 42.81 c = 64.92

Topic IV: Circles Connected with Triangle:

10. With usual notation prove that $r_2 + r_2 + r_3 - r = 4R$ **(2 times)**
11. Prove that$(r_1 + r_2)$ than $\frac{\gamma}{2}$= c. **(3 times)**
12. Prove that in an equilateral triangle $r : R : r_1 : r_2 : r_3 = 1 : 2 : 3 : 3 : 3$
13. Prove that $\frac{1}{r^2} + \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} = \frac{a^2+b^2+c^2}{\Delta^2}$ **(3 times)**
14. Show that $r = 4R \sin\frac{\alpha}{2} \sin\frac{\beta}{2} \sin\frac{\gamma}{2}$. **(2 times)**
15. With usual notation show that: $r_1 = 4R \sin\frac{\alpha}{2} \cos\frac{\beta}{2} \cos\frac{\gamma}{2}$. **(2 times)**
16. Prove that $\Delta = 4R\, r \cos\frac{\alpha}{2} \cos\frac{\beta}{2} \cos\frac{\gamma}{2}$ **(2 times)**
17. Prove that in an equilateral triangle, $r : R : r_1 = 1 : 2 : 3$ **(2 times)**
18. Solve the triangle ABC if $a=53;\ \beta=88°36;\ \gamma=31°54'$
19. Solve the triangle ABC in which $a=\sqrt{3}-1, b=\sqrt{3}+1\, and\, \gamma=60°$
20. Prove that $abc(\sin\alpha+\sin\beta+\sin\gamma)=4\Delta\gamma$
21. P and Q are two points in line with If the distance between P and Q be 30m and the angles of elevation of the top of the tree at P and C be 12° and 150 respectively, find the height of the tree.
22. Solve the triangle ABC if b = 61; a = 32 and $\alpha = 59°30'$ using first law of tangents and then law of sines.
23. Show that $r_3 = 4R\cos\frac{\alpha}{2}.\cos\frac{\beta}{2}.\sin\frac{\gamma}{2}$

OBJECTIVE MCQ'S OF CHAPTER-13 IN ALL PUNJAB BOARDS 2011-2021

1. $Tan^{-1}A + Tan^{-1}B =$ ______: (3 times)
(A) $Tan^{-1}\frac{A-B}{1+AB}$ (B) $Tan^{-1}\frac{A+B}{1+AB}$ (C) $Tan^{-1}\frac{A+B}{1-AB}$ (D) $Tan^{-1}\frac{A-B}{1-AB}$

2. $Sin^{-1}(A\sqrt{1-B^2} + B\sqrt{1-A^2})$ (3 times)
(A) $Sin^{-1}A + Sin^{-1}B$ (B) $Sin^{-1}A - Sin^{-1}B$ (C) $Cos^{-1}A + Cos^{-1}B$ (D) $Cos^{-1}A - Cos^{-1}B$

3. Y = Sinx is called Principal Sine function for restricted domain: (2 times)
(A) $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ (B) $0 \le x \le \frac{\pi}{2}$ (C) $0 \le x \le \pi$ (D) $< x < \frac{\pi}{2}$

4. $Cos\left(Sin^{-1}\frac{1}{\sqrt{2}}\right)$ is equal to: (6 times)
(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{-\pi}{4}$

5. The domain of the Principal Tan Function is: (2 times)
(A) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (B) $(0,\pi)$ (C) R (D) $(-\pi, \pi)$

6. The value of $Cos(tan^{-1}0)$ is equal to: (1 time)
(A) 0 (B) 1 (C) -1 (D) $\frac{1}{2}$

7. Domain of y = $Co\sec^{-1}$ is : (4 times)
(A) $x \le -1$ or $x \ge 1$ (B) $\left(\frac{-\pi}{2}, \frac{\pi}{4}\right)$ (C) R (D) $-1 \le x \le 1$

8. The domain of the principal sine function is:- (4 times)
(A) $\left[0, \frac{\pi}{2}\right]$ (B) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (C) $\left[0, \frac{3\pi}{2}\right]$ (D) $[0, 2\pi]$

9. $Sin^{-1}A + Sin^{-1}B =$ (6 times)
(A) $Cos\text{-}1[AB + \sqrt{1-A^2}.\sqrt{1-B^2}]$ (B) $Sin^{-1}[A\sqrt{1-B^2} + B\sqrt{1-A^2}]$
(C) $Cos\text{-}1\ [2A-1]$ (D) $Cos^{-1}[A\sqrt{1-B^2} - B\sqrt{1-A^2}]$

10. Range of $Cos^{-1}x$ is: (5 times)
(A) $[0, \pi]$ (B) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (C) $[-1, 1]$ (D) R

11. Principle domain of cos x is (2 times)
(A) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (B) $\left(0, \frac{\pi}{2}\right)$ (C) $[-\pi, \pi]$ (D) $[0, \pi]$

12. $Tan(tan^{-1}(-1))$ equal: (4 times)
(A) 1 (B) $\frac{\pi}{4}$ (C) $-\frac{\pi}{4}$ (D) -1

13. $Tan^{-1}(-1) =$ (5 times)
(A) $\pi/6$ (B) $-\pi/4$ (C) $\pi/2$ (D) $-\pi$

14. $2Tan^{-1}A =$ ______ (5 times 2018)
(A) $tan^{-1}\left(\frac{A}{1-A^2}\right)$ (B) $tan^{-1}\left(\frac{2A}{1+A^2}\right)$ (C) $tan^{-1}\left(\frac{2A}{1-A^2}\right)$ (D) $tan^{-1}\left(\frac{-2A}{1-A^2}\right)$

15. $sin^{-1}0 = \beta$, then value of β is: (4 times)
(A) 0 (B) 1 (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

16. $tan(sin^{-1}x)$ is equal to:
(A) $1 + 2x^2$ (B) $1-x^2$ (C) $\frac{x}{\sqrt{1-x^2}}$ (D) $\frac{2x}{\sqrt{1+x^2}}$

17. $Tan^{-1}(-1)$ is equal to =
(A) $\frac{\pi}{4}$ (B) $-\frac{\pi}{4}$ (C) $\cdot\frac{\pi}{2}$ (D) $\frac{\pi}{2}$

18. $tan^{-1}\left(\frac{A-B}{1+AB}\right)$ equals to:
(A) $tan^{-1}A + tan^{-1}B$ (B) $cot^{-1}A + cot^{-1}B$ (C) $tan^{-1}A, - tan^{-1}B$ (D) $tan^{-1}A - tan^{-1}B$

19. $cos^{-1}(-x) =$: (2 times 2018)
(A) $\pi - cos^{-1}x$ (B) $cos^{-1}x$ (C) $\frac{\pi}{2} - cos^{-1}x$ (D) $\pi + cos^{-1}x$

20. $\sin^{-1}x$ is equal to:
(A) $\frac{\pi}{2}+\sin^{-1}x$ (B) $\frac{\pi}{2}-\sin^{-1}x$ (C) $\frac{\pi}{2}+\cos^{-1}x$ (D) $\frac{\pi}{2}-\cos^{-1}x$

21. $\sin\left(\sin^{-1}(\frac{1}{2})\right)$ is equal to:
(A) $\pi/3$ (B) $\pi/6$ (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$

22. Domain of y = principal sin x is:
(A) R (B) [-1,1] (C) $[0,\pi]$ (D) $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

23. $\sin^{-1}(-1/2)=$
(A) $\pi/3$ (B) $-\pi/6$ (C) $\pi/4$ (D) $-\pi/3$

24. $\sec\left(\cos^{-1}\frac{1}{2}\right)=$ ____________
(A) 2 (B) $\frac{1}{2}$ (C) 1 (D) $\sqrt{2}$

25. $\tan^{-1}(-\sqrt{3})=$ __________ **(2 times)**
(A) $\frac{2\pi}{3}$ (B) $\frac{-2\pi}{3}$ (C) $\frac{-\pi}{6}$ (D) $\frac{+\pi}{3}$

26. $\cos^{-1}(-x)=$
(A) $-\cos^{-1}x$ (B) $\cos^{-1}x$ (C) $\pi-\cos^{-1}x$ (D) $\frac{\pi}{2}-\cos^{-1}x$

27. $\cos^{-1}(-x)$ is equal to: **(2 times)**
(A) $\cos^{-1}x$ (B) $\pi+\cos^{-1}x$ (C) $\pi-\cos^{-1}x$ (D) $\sin^{-1}x$

28. $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=$: **(2 times 2018)**
(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

29. The value of $Sec\left(Sin^{-1}\frac{\sqrt{3}}{2}\right)=$ **(2 times)**
(A) $\frac{1}{2}$ (B) 2 (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{2}}$

30. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is equal to:
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$

31. $\cos(\tan^{-1}\sqrt{3})=$ **(2 times)**
(A) $\frac{1}{2}$ (B) $\frac{-1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{-\sqrt{3}}{2}$

32. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=$:
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$

33. $\cos(\sec^{-1}1)$ is equal to:
(A) 1 (B) 0 (C) 30° (D) 2

34. $\cos(\sin^{-1}\frac{1}{2})$ equals:
(A) $-\frac{\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$

35. $\sec(\cos^{-1}1/2)$ is equal to: **(2 times)**
(A) $1/2$ (B) 60° (C) 30° (D) 2

36. $2\tan^{-1}A$ equals: **(2 times)**
(A) $\tan^{-1}\left(\frac{A}{1-A^2}\right)$ (B) $\tan^{-1}\left(\frac{A}{1+A^2}\right)$ (C) $\tan^{-1}\left(\frac{-2A}{1+A^2}\right)$ (D) $\tan^{-1}\left(\frac{2A}{1-A^2}\right)$

37. $\sin\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$ equals:
(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$

38. $\tan^{-1}(-1)=$
(A) $\frac{\pi}{6}$ (B) $-\pi$ (C) $\frac{\pi}{4}$ (D) $-\frac{\pi}{4}$

39. $\text{Sec}\left(\text{Sin}^{-1}\frac{\sqrt{3}}{2}\right) =$:

(A) $\frac{1}{2}$ (B) 2 (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{2}}$

2018

40. The value of $\frac{\pi}{2} - \sin^{-1}x$ Is equal to.

(a) $\cos^{-1}x$ (b) $\sin^{-1}x$ (c) $\cos x$ (d) $\sin x$

41. $\cos^{-1}(-x)$ Is equal to:

(a) $\frac{\pi}{2} - \sin^{-1}x$ (b) $\frac{\pi}{2} + \sin^{-1}x$ (c) $\pi + \sin^{-1}x$ (d) $\pi - \cos^{-1}x$

42. The value of $\sin(\tan^{-1}(0)) =$

(a) 0 (b) 1 (c) -1 (d) ∞

2019

43. $\text{Tan}^{-1}(1) =$:

(a) $\pi/3$ (b) $\pi/4$ (c) $\pi/6$ (d) π

44. $\text{Sin}(\text{Tan}^{-1}0^0) =$:

(a) -1 (b) 1 (c) 0 (d) ∞

45. The value of $\sin\left(\cos^{-1}\frac{\sqrt{3}}{2}\right)$ is equal to:

(a) 1 (b) -1 (c) $\frac{-1}{2}$ (d) $\frac{1}{2}$

46. $\tan\left(\tan^{-1}(1)\right) =$:

(a) 1 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) 0

47. $\cos^{-1}(-x) =$:

(a) $-\cos^{-1}x$ (b) $\cos^{-1}x$ (c) $\pi - \cos^{-1}x$ (d) $\frac{\pi}{2} - \cos^{-1}x$

48. The value of $\sin^{-1}(\cos\pi/6)$ is

(a) $\pi/6$ (b) $\pi/2$ (c) $\frac{3\pi}{2}$ (d) $\pi/3$

ANSWERS TO THE MULTIPLE CHOICE QUESTIONS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
C	A	A	A	A	B	A	B	B	A	D	D	B	C	A
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
C	B	C	A	D	D	D	B	A	A	C	C	B	B	B
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
A	B	A	B	D	D	C	D	B	A	D	A	B	C	D
46	47	48												
A	C	D												

SHORT QUESTION'S OF CHAPTER-13 IN ALL PUNJAB BOARDS 2011-2021

1. Find the value of $\text{Sec}\left[\text{Sin}^{-1}\left(\frac{-1}{2}\right)\right]$ without using table / calculator. (5 times)

Sol. $\text{Sec}\left(\text{Sin}^{-1}(-1/2)\right)$

Let. $\sin^{-1}\left(-\frac{1}{2}\right) = A \longrightarrow (1)$

$\therefore \sec\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) = \sec A \longrightarrow (2)$

By (1) $\sin A = 1/2$

$\cos^2 A = 1 - \sin^2 A$

$\cos^2 A = 1 - \left(-\frac{1}{2}\right)^2$

$\cos^2 A = 1 - 1/4$

$\cos^2 A = \frac{4-1}{4} = \frac{3}{4}$

$\cos A = \frac{\sqrt{3}}{2}$

$\sec A = \frac{2}{\sqrt{3}}$

Thus by (2)

$\sec\left(\sin^{-1}(-1/2)\right) = 2/\sqrt{3}$

2. Find the value of Sin $\left(\cos^{-1}\frac{\sqrt{3}}{2}\right)$ **(2 times 2018)**

Sol. $\sin\left(\cos^{-1}\frac{\sqrt{3}}{2}\right) \rightarrow (1)$

Let $\cos^{-1}\frac{\sqrt{3}}{2} = y$

$\cos y = \frac{\sqrt{3}}{2}$ $\quad \because \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\cos y = \cos\frac{\pi}{6}$

$\Rightarrow y = \frac{\pi}{6}$

so. $\sin\left(\cos^{-1}\frac{\sqrt{3}}{2}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

3. Show that : $\cos^{-1}(-x) = \pi - \cos^{-1}x$. **(2 times)**

Sol. $\cos^{-1}(-x) = \pi - \cos^{-1}x$

$\cos^{-1}(-x) + \cos^{-1}x = \pi$

Now. L.H.S $\cos^{-1}(-x) + \cos^{-1}x$

$\because \cos^{-1}A + \cos^{-1}B = \cos^{-1}\left(AB - \sqrt{(1-A^2)(1-B^2)}\right)$

$= \cos^{-1}\left(-x.x - \sqrt{1-(-x)^2(1-x^2)}\right)$

$= \cos^{-1}\left(-x^2 - \sqrt{(1-x^2)(1-x^2)}\right)$

$= \cos^{-1}\left(-x^2 - \sqrt{(1-x)^2}\right) \Rightarrow \cos^{-1}(-x^2 - (1-x^2))$

$= \cos^{-1}(-x^2 - 1 + x^2) \Rightarrow \cos^{-1}(-1)$

$= \pi = $ R.H.S

4. Show that $\sin(2\cos^{-1}x) = 2x\sqrt{1-x^2}$. **(4 times)**

Sol. $\sin(2\cos^{-1}x) = 2x\sqrt{1-x^2}$

L.H.S = $\sin(2\cos^{-1}x)$

Let $\cos^{-1}x = \theta$

$x = \cos\theta \Rightarrow \sin 2\theta = 2\sin\theta\cos\theta = 2\cos\theta\sqrt{1-\cos^2\theta}$

$= 2x\sqrt{1-x^2} =$ R.H.S

5. Write the range of $\sin^{-1}x$.

Sol. The range of $\sin^{-1}x$ is $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

6. Show that $\cos(\sin^{-1}x) = \sqrt{1-x^2}$

Sol. $\cos(\sin^{-1}x) = \sqrt{1-x^2}$

L.H.S $= \cos(\sin^{-1}x)$ Let $\sin^{-1}x = A$ ....(1)

$\Rightarrow \quad x = \sin A$ ....(2)

$\cos A = \cos A$

$\cos A = \sqrt{1 - \sin^2 A} \qquad = \sqrt{1 - x^2}$

By (1) = R.H.S Proved

7. Evaluate without using table $\sin^{-1}(1)$.

Sol. $\sin^{-1}(1)$

Let $y = \sin^{-1}(1)$

$\Rightarrow \quad \sin y = 1 \qquad \therefore y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\therefore \quad y = \frac{\pi}{2}$

Thus $\sin^{-1}(1) = \frac{\pi}{2}$

8. Without using calculator show that $\tan^{-1}\frac{5}{12} = \sin^{-1}\frac{5}{13}$. **(2 times)**

Sol. $\tan^{-1}\frac{5}{12} = \sin^{-1}\frac{5}{13}$

Let $\sin^{-1}\frac{5}{13} = A \qquad \rightarrow (1)$

$\Rightarrow \quad \frac{5}{13} = \sin A$

$\cos A = \sqrt{1 - \sin^2 A}$

$= \sqrt{1 - \left(\frac{5}{13}\right)^2} = \sqrt{1 - \frac{5}{169}} = \sqrt{\frac{169-5}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$

$\cos A = \frac{12}{13}$

$\tan A = \frac{\sin A}{\cos A} = \frac{5/13}{12/13} = \frac{5}{12}$

$\Rightarrow \quad A = \tan^{-1}\frac{5}{12} \qquad \rightarrow (2)$

By (1) and (2)

$\tan^{-1}\frac{5}{12} = \sin^{-1}\frac{5}{13}$

9. Show that $\cos(\sin^{-1}x) = \sqrt{1 - x^2}$ **(7 times)**

Sol. Let $\sin^{-1} = \theta \Rightarrow x = \sin\theta$

We know $\cos^2\theta = 1 - \sin^2\theta = 1 - x^2 = \sqrt{1 - x^2}$ Proved

10. Find the value of $\sin(\tan^{-1}(-1))$

Sol. $\sin(\tan^{-1}(-1)$

$= \sin(-45^\circ) \qquad \tan^{-1}(-1) = -45^\circ$

$= -\sin 45^\circ \qquad \sin(-\theta) = -\sin\theta$

$= \frac{-1}{\sqrt{2}}$

11. Find the value of $\tan\left(\cos^{-1}\frac{\sqrt{3}}{2}\right)$ **(2 times)**

Sol. $\tan\left(\cos^{-1}\frac{\sqrt{3}}{2}\right) \qquad \cos^{-1}\frac{\sqrt{3}}{2} = 30^\circ$

$= \tan(30^\circ) = \tan 30^\circ = \frac{1}{\sqrt{3}}$

12. Show that $\tan(\sin^{-1}x) = \frac{x}{\sqrt{1-x^2}}$ **(3 times)**

Sol. Let $\sin^{-1}x = \theta \Rightarrow x = \sin\theta$

L.H.S $= \tan(\sin^{-1}x)$

$= \tan(\theta) = \frac{\sin\theta}{\cos\theta} = \frac{\sin\theta}{\sqrt{\cos^2\theta}} = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}} = \frac{x}{\sqrt{1-x^2}}$ = R.H.S

13. Show that $\tan^{-1}\frac{5}{12} = \sin^{-1}\frac{5}{13}$

Sol. Let $\tan^{-1}\frac{5}{12} = \theta \Rightarrow \frac{5}{12} = \tan\theta$

OR $\cot\theta = \frac{12}{5}$

We know

$Cosec^2\theta = 1 + Cot^2\theta = 1 + \left(\frac{12}{5}\right)^2 = 1 + \frac{144}{25} = \frac{25+144}{25}$

$Cosec^2\theta = \frac{169}{25} = \frac{25}{169}$

$Sin\,\theta = \frac{5}{13}$

$\theta \quad = Sin^{-1}\frac{5}{13}$

$Tan^{-1}\frac{5}{12} = Sin^{-1}\frac{5}{13}$ Proved

14. **Show that $Sin^{-1}(-x) = -Sin^{-1}x$** **(4 times)**

Sol $Sin^{-1}(-x) = -Sin^{-1}x$

$Sin^{-1}(-x) + Sin^{-1}x = 0$

L.H.S $= Sin^{-1}(-x) + Sin^{-1}x = 0$

We Know

$\because Sin^{-1}A + Sin^{-1}B = Sin^{-1}\left(A\sqrt{1-B^2} + B\sqrt{1-A^2}\right)$

$Sin^{-1}(x) + Sin^{-1}(-x) = Sin^{-1}\left(x\sqrt{1-x^2} + x\sqrt{1-x^2}\right)$

$Sin^{-1}(x) + Sin^{-1}(-x) = Sin^{-1}(0)$

$Sin^{-1}x + Sin^{-1}(-x) = 0$

$Sin^{-1}(-x) = -Sin^{-1}x$ Proved

15. **Show that $Cos(2Sin^{-1}x) = 1 - 2x^2$** **(2 times)**

Sol. Let $Sin^{-1}x = \theta \Rightarrow x = Sin\,\theta$

$\therefore Cos(2Sin^{-1}x) = Cos(2\theta)$

$= Cos^2\theta - Sin^2\theta = 1 - Sin^2\theta - Sin^2\theta$

$Cos(2Sin^{-1}x) = 1 - 2sin^2\theta$

$Cos(2Sin^{-1}x) = 1 - 2x^2$ Proved

16. **Find the value of $Cos\left(Sin^{-1}\frac{1}{\sqrt{2}}\right)$** **(2 times)**

Sol. $Cos\left(Sin^{-1}\frac{1}{\sqrt{2}}\right)$

$= Cos(45^\circ) = \frac{1}{\sqrt{2}}$

17. **Without using calculator, show that** $Cos^{-1}\frac{4}{5} = Cot^{-1}\frac{4}{3}$

Sol $Cos^{-1}\frac{4}{5} = Cot^{-1}\frac{4}{3}$

L.H.S. $= Cos^{-1}\frac{4}{5}$

$= \theta$

$= Cot^{-1}(Cot\theta)$

$= Cot^{-1}\left(\frac{Cos\theta}{Sin\theta}\right)$

$= Cot^{-1}\left(\frac{4/5}{3/5}\right)$

$= Cot^{-1}\frac{4}{3}$

$= R.H.S$

Let $\theta = Cos^{-1}\frac{4}{5}$

$Cos\theta = \frac{4}{5}$

$Sin\theta = \sqrt{1 - Cos^2\theta}$

$= \sqrt{1 - \left(\frac{4}{5}\right)^2}$

$= \sqrt{1 - \frac{16}{25}}$

$= \sqrt{\frac{25-16}{25}}$

$Sin\theta = \sqrt{\frac{9}{25}} = \frac{3}{5}$

L.H.S = R.H.S

18. **Find domain and range of $y = \cos^{-1}x$.** **(2 times 2018)**

Sol Domain and of $y=\cos^{-1}x$ is

Domain $= -1 \le x \le 1$ Range $= 0 \le x \le \pi$

19. **What is the domain and range of $\sin^{-1}x$?** **(2 times 2018)**

Sol Domain and range of Sinx

Domain $= -\frac{\pi}{2} \le x \le \frac{\pi}{2}$ Range $= -1 \le x \le 1$

2019

20: **Prove that** $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5} = \tan^{-1}\frac{9}{19}$

Sol: L.H.S = $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5}$

$$\because \tan^{-1}A + \tan^{-1}B = \tan^{-1}\frac{A+B}{1-AB}$$

$$= \tan^{-1}\left[\frac{\frac{1}{4}+\frac{1}{5}}{1-\left(\frac{1}{4}\right)\left(\frac{1}{5}\right)}\right]$$

$$= \tan^{-1}\left(\frac{\frac{5+4}{20}}{1-\frac{1}{20}}\right) = \tan^{-1}\left(\frac{\frac{9}{20}}{\frac{20-1}{20}}\right) = \tan^{-1}\left(\frac{\frac{9}{20}}{19/20}\right)$$

$$= \tan^{-1}\left(\frac{9}{20}\times\frac{20}{19}\right) = \tan^{-1}\frac{9}{19} = R.H.S$$

L.H.S = R.H.S

21: **Find the value of** $Cos\left(Sin^{-1}\frac{1}{2}\right)$

Sol: $Cos\left(Sin^{-1}\frac{1}{2}\right)$

$= Cos30^{\circ}$ $\because Sin^{-1}\frac{1}{2} = 30^{0}$

$= \frac{\sqrt{3}}{2}$ $\because Cos30^{0} = \frac{\sqrt{3}}{2}$

22: **Find the value of** $Sec\left(Cos^{-1}\frac{1}{2}\right)$

Sol: Let $y = Cos^{-1}\frac{1}{2}$ ________(i)

$\Rightarrow Cosy = \frac{1}{2} \Rightarrow y = \frac{\pi}{3} \Rightarrow Cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ (Using i)

Now $Sec\left(Cos^{-1}\frac{1}{2}\right) = Sec\frac{\pi}{3} = \frac{1}{Cos\frac{\pi}{3}} = \frac{1}{1/2} = 2$

23: **Prove that** $Sin^{-1}x = \frac{\pi}{2} - Cos^{-1}x$

Sol: $Sin^{-1}x=\frac{\pi}{2}-\cos^{-1}x$

Let $\alpha=\frac{\pi}{2}-\cos^{-1}x$

$\Rightarrow \cos^{-1}x=\frac{\pi}{2}-\alpha$

$\Rightarrow x=\cos\left(\frac{\pi}{2}-\alpha\right)$

$\because \cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$

$x=\sin\alpha$

$\Rightarrow \sin^{-1}x=\alpha$

Put the value of α

$\sin^{-1}x=\frac{\pi}{2}-\cos^{-1}x$

2021

24- Find the value of cosec(tan⁻¹ (- 1))

Sol: $\cos ec^2\left(\tan^{-1}(-1)\right)$

$=\cos ec\left(-\tan^{-1}1\right)$ $\because \tan 45^o=1$

$=\cos ec\left(-45^o\right)$ $\because 45^o=\tan^{-1}1$

$=-\cos ec 45^o$ $\because \cos ce(-\theta)=-\cos ec\theta$

$=\frac{-1}{\sin 45^o}$

$=\frac{-1}{\frac{1}{\sqrt{2}}}$ $\because \sin 45^o=\frac{1}{\sqrt{2}}$

$=-\sqrt{2}$

25- Without using tables/calculator , Find cos⁻¹ $\left(\frac{\sqrt{3}}{2}\right)$

Sol: Let $\alpha=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$........(1)

$\Rightarrow \cos\alpha=\frac{\sqrt{3}}{2}$, $0\le\alpha\le\pi$

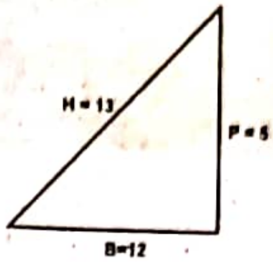
$\Rightarrow \alpha=\frac{\pi}{6}$ $\because \cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

Hence $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{6}$ *from eq(1)*

26- Without using table/calculator Prove that $\tan^{-1}(5/12)=\sin^{-1}(5/13)$

Sol: Let $\alpha=\tan^{-1}\frac{5}{12}$........(1)

$\Rightarrow \tan\alpha=\frac{5}{12}$, $\frac{-\pi}{2}<\alpha<\frac{\pi}{2}$



By Pythagoras theorem

$H^2 = B^2 + P^2 \quad \Rightarrow \quad H^2 = (12)^2 + (5)^2$

$H^2 = 144 + 25 \quad \Rightarrow \quad H^2 = 169$

$H = 13$

So, $\sin\alpha = \dfrac{P}{H}$

$\sin\alpha = \dfrac{5}{13} \qquad = \sin^{-1}\dfrac{5}{13}$ ....(2)

Comparing (1) and (2)

$\tan^{-1}\dfrac{5}{12} = \sin^{-1}\dfrac{5}{13}$

Hence prove it.

LONG QUESTION'S OF CHAPTER-13 IN ALL PUNJAB BOARDS 2011-2021

1. Prove that $2\text{Tan}^{-1}\frac{1}{3} + \text{Tan}^{-1}\frac{1}{7} = \frac{\pi}{4}$. (6 times)
2. Show that $\text{Sin}^{-1}\frac{77}{85} - \text{Sin}^{-1}\frac{3}{5} = \text{Cos}^{-1}\frac{15}{17}$ without using calculator. (5 times)
3. Prove that $\text{Tan}^{-1}\frac{3}{4} + \text{Tan}^{-1}\frac{3}{5} - \text{Tan}^{-1}\frac{8}{19} = \frac{\pi}{4}$ (9 times)
4. Prove that $\text{Sin}^{-1}\frac{1}{\sqrt{5}} + \text{Cot}^{-1} 3 = \frac{\pi}{4}$. (8 times)
5. Prove that $\text{Sin}^{-1}\frac{5}{13} + \text{Sin}^{-1}\frac{7}{25} = \text{Cos}^{-1}\frac{253}{325}$ (8 times)
6. Prove that $\text{Sin}^{-1}\frac{3}{5} + \text{Sin}^{-1}\frac{8}{17} = \text{Sin}^{-1}\frac{77}{85}$. (3 times)
7. Find the value of $\cos^{-1}\frac{4}{5}$ without using calculator.
8. Prove that $\cos^{-1}\frac{63}{65} + 2\tan^{-1} = \frac{1}{5} = \sin^{-1}\frac{3}{5}$. (2 times)
9. Prove that $\cos^{-1} A + \cos^{-1} B = \cos^{-1}\left(AB - \sqrt{(1-A^2)(1-B^2)}\right)$
10. Prove that $\tan^{-1}\dfrac{120}{119} = 2\cos^{-1}\dfrac{12}{13}$
11. Prove that $\tan^{-1}\dfrac{1}{11} + \tan^{-1}\dfrac{5}{6} = \tan^{-1}\dfrac{1}{3} + \tan^{-1}\dfrac{1}{2}$
12. Show that $2\tan^{-1}\dfrac{2}{3} = \sin^{-1}\dfrac{12}{13}$ (2 Times)

OBJECTIVE MCQ'S OF CHAPTER-14 IN ALL PUNJAB BOARDS 2011-2021

1. If Sin x = $\frac{\sqrt{3}}{2}$ then reference angle is: (5 times)
(A) $\pi/6$ (B) $\pi/3$ (C) $-\pi/3$ (D) $-\pi/6$

2. If cos x = $\frac{1}{2}$ then reference angle is: (8 times)
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

3. The solution set of Tan2x = 1 in $[0,2\pi]$ is: (4 times)
(A) $\left\{\frac{\pi}{8},\frac{5\pi}{8}\right\}$ (B) $\left\{\frac{\pi}{4},\frac{5\pi}{4}\right\}$ (C) $\left\{\frac{\pi}{4},\frac{3\pi}{4}\right\}$ (D) $\left\{\frac{\pi}{6},\frac{3\pi}{6}\right\}$

4. The domain of principal Cos function is: (5 times)
(A) -1 (B) 1 (C) 0 (D) $[0,\pi]$

5. If Sin x = $\frac{1}{2}$; then reference angle is: (3 times)
(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{-\pi}{6}$

6. If cos x = $-\frac{1}{2}$ then the reference angle is: (3 times)
(A) $\frac{\pi}{3}$ (B) $-\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $-\frac{\pi}{6}$

7. Solution of 1 + Cos x = 0 is: (5 times)
(A) $\frac{\pi}{2}$ (B) π (C) 2π (D) $\frac{\pi}{6}$

8. General solution of Sinx = 0 is: (7 times)
(A) $\left\{0,\frac{\pi}{2}\right\}$ (B) $x = n\pi$ (C) $x = \frac{n\pi}{2}$ (D) $x = \frac{(2n+1)\pi}{2}$

9. If tan x = -1 then x = (5 times)
(A) $\pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$ (B) $\frac{\pi}{4}, \pi + \frac{\pi}{4}$ (C) $\frac{\pi}{4}, \frac{-\pi}{4}$ (D) $\pi - \frac{\pi}{4}, \pi + \frac{\pi}{4}$

10. If secx = -2, then reference angle of sec x is:- (3 times)
(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $-\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

11. If Cos2x = 0 , then solution in 1st quadrant is: (5 times)
(A) 30° (B) 45° (C) 60° (D) 15°

12. An equation containing at least one trigonometric function is called:-
(A) Algebric equation (B) Quadratic equation (C) Linear equation (D) Trigonometric equation

13. The solutions of equation $\frac{1}{2} + sin\theta$ are in quadrant.
(A) 1 & IV (B) I & III (C) III & IV (D) II & IV

14. Solution of the equation Cosx = -1 in $[0, 2\pi]$ is:
(A) $\{0, \pi\}$ (B) $\{0, 2\pi\}$ (C) $\{\pi\}$ (D) $\{0\}$

15. If sin x = $\frac{1}{2}$ then x =
(A) $\frac{\pi}{6}, \frac{5\pi}{6}$ (B) $\frac{-\pi}{6}, \frac{5\pi}{6}$ (C) $\frac{-\pi}{6}, \frac{-5\pi}{6}$ (D) $\frac{\pi}{3}, \frac{2\pi}{3}$

16. If Sin x = Cos x then x = (3 times 2018)
(A) 45° (B) 30° (C) 0° (D) 60°

17. cos x = $\frac{1}{2}$ has solution:
(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

18. The solution of equation cos x = -1 in $[0, 2\pi]$ is: (2 times)
(A) π (B) $\frac{-\pi}{2}$ (C) 2π (D) $\frac{\pi}{2}$

19. Solution set of tan2x = 1 in $[0, 2\pi]$ is equal to:
(A) $\{\pi/4, 5\pi/4\}$ (B) $\{\pi/8, 5\pi/8\}$ (C) $\{\pi/4, 3\pi/4\}$ (D) $\{\pi/5, 5\pi/6\}$

20. If $\cos x = -\frac{\sqrt{3}}{2}$ then reference angle of cos x is:
(A) $\frac{\pi}{6}$ (B) $-\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{5\pi}{6}$

21. Solutions of the equation $1 + \cos\theta = 0$ are in quadratns:
(A) I and IV (B) II and III (C) II and IV (D) None of these

22. Solution of $1+\cos x = 0$ in $[0.2\pi]$
(A) $x = 0$ (B) $x = \pi$ (C) $x = \frac{\pi}{2}$ (D) $x = \frac{\pi}{3}$

23. An equation containing at least one trigonometric function is called
(A) algebraic equation (B) equation (C) linear equation (D) trigonometric equation

24. Reference angle of $\tan\theta = -1$ equals
(A) $\frac{\pi}{4}$ (B) $\frac{-\pi}{4}$ (C) $-\pi$ (D) $\frac{\pi}{2}$

25. Solution of equation $\tan x = \frac{1}{\sqrt{3}}$ is : **(2 times)**
(A) I & III quad (B) I&II quad (C) II &IV quad (D) I quad

26. The Trigonometric Equation has solution :
(A) 0 (B) 1 (C) 2 (D) Infinite

27. $\cos x = \frac{-\sqrt{3}}{2}$ then Reference angle of Cos x is:
(A) $\frac{\pi}{6}$ (B) $-\frac{\wedge}{6}$ (C) $\frac{\wedge}{3}$ (D) $\frac{5\wedge}{6}$

28. $\cos x = \frac{1}{2}$ has solution ---------- $x \in [0, \pi]$:
(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

29. An equation containing at least one trigonometric function is called:
(A) Algebraic equation (B) Quadratic equation (C) Linear equation (D) Trigonometric equation

30. If $\sin x = \frac{1}{2}$ then x equals , $x \in [0, 2\pi]$ **(2 times)**
(A) $\frac{-\pi}{6}, \frac{-5\pi}{6}$ (B) $\frac{-\pi}{6}, \frac{5\pi}{6}$ (C) $\frac{\pi}{2}, \frac{2\pi}{3}$ (D) $\frac{\pi}{6}, \frac{5\pi}{6}$

31. If $n \in Z$, the General Solution of the equation $\sin x = 0$ is:
(A) $\left\{\frac{n\pi}{2}\right\}$ (B) $\left\{\frac{n\pi}{3}\right\}$ (C) $\left\{\frac{n\pi}{4}\right\}$ (D) $\{n\pi\}$

32. If $\cos x = \frac{\sqrt{3}}{2}$, $x \in [0, \pi]$, then x equals:
(A) $\frac{-\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) $\frac{\pi}{6}$ (D) $\frac{7\pi}{6}$

33. If $\sin x = -\frac{\sqrt{3}}{2}$, then reference angle is:
(A) $\frac{\pi}{6}$ (B) $-\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $-\frac{\pi}{3}$

34. If $\cos 2x = 0$ solution in 1st quadrant is:
(A) 30° (B) 60° (C) 45° (D) 15°

35. If $\sin x = \cos x$ then x =:
(A) 45° (B) 30° (C) 0° (D) 60°

2018

36. An equation containing at least one trigonometric function is called.
(a) Algebraic Equation (b) Quadratic equation (c) Linear equation (d) Trigonometric equation

37. The solution $\sin x + \cos x = 0$ in $[0, \pi]$ is
(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

38. If $\sin x = \frac{\sqrt{3}}{2}$ and $x \in [0, 2\pi]$ then x is:
(a) $\frac{5\pi}{3}, \frac{4\pi}{3}$ (b) $\frac{\pi}{4}, \frac{3\pi}{4}$ (c) $\frac{\pi}{3}, \frac{2\pi}{3}$ (d) $\frac{\pi}{6}, \frac{5\pi}{6}$

(2 times)

39. If $\sin x = \frac{\sqrt{3}}{2}$ and $x \in [0, 2\pi]$ then x is:
(a) $\frac{5\pi}{3}, \frac{4\pi}{3}$ (b) $\frac{\pi}{4}, \frac{3\pi}{4}$ (c) $\frac{\pi}{3}, \frac{2\pi}{3}$ (d) $\frac{\pi}{6}, \frac{5\pi}{6}$

40. If $\cos x = \frac{1}{\sqrt{2}}$, then reference angle is:

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

41. Solution of the equation $\tan x + 1 = 0$ is:

(a) $\{\frac{3\pi}{4} + n\pi\}$ (b) $\{\frac{\pi}{4} + n\pi\}$

(c) $\{\pi + n\pi\}$ (d) $\{2\pi + n\pi\}$, when $n \in z$.

42. Solution of equation cos x + 1 = 0 is: (2 times)

(a) $\{\pi + n\pi\}$ (b) $\{\pi + 2n\pi\}$ (c) $\{\pi\}$ (d) $\left\{\frac{\pi}{2} + n\pi\right\}$

2019

43. $\cos x = \frac{1}{2}$ has a solution:

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

44. The reference angle for $\text{Tan}\,\theta = \sqrt{3}$ is:

(a) $\frac{\pi}{6}$ (b) $-\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{-\pi}{3}$

45. The solution of cosecθ = 2 in interval [0, 2π] is equal to:

(a) $\frac{\pi}{6}, \frac{7\pi}{6}$ (b) $\frac{\pi}{6}, \frac{5\pi}{6}$ (c) $\frac{\pi}{3}, \frac{5\pi}{6}$ (d) $\frac{\pi}{3}, \frac{\pi}{6}$

46. Solution of $\cot\theta = \frac{1}{\sqrt{3}}$ in IIIrd quadrant is:

(a) $\frac{5\pi}{4}$ (b) $\frac{7\pi}{4}$ (c) $\frac{4\pi}{3}$ (d) π

47. Solution of equation $\tan x = \frac{1}{\sqrt{2}}$ lies in the quadrants:

(a) I and II (b) II and III (c) I and III (d) I and IV

48. The solution of $\tan x = \frac{1}{\sqrt{3}}$ for $x \in [0, \pi]$ is

(a) $\{\pi/2\}$ (b) $\{\pi/6\}$ (c) $\{\pi/3\}$ (d) $\{\pi/4\}$

49. The solutions of equation $1 + \sin\theta = 0$ are in quadrant

(a) I and IV (b) I and III (c) II and IV (d) III and IV

2021

50. If $\sin x = \frac{-\sqrt{3}}{2}$, then solution is:

(a) $\frac{4\pi}{6}, \frac{5\pi}{6}$ (b) $\frac{4\pi}{3}, \frac{5\pi}{3}$ (c) $\frac{5\pi}{6}, \frac{7\pi}{6}$ (d) $\frac{\pi}{3}, \frac{7\pi}{3}$

51. If cosx = 0 the number of solutions are

(a) 2 (b) 4 (c) 6 (d) infinite

52. If $\sin x = \frac{-1}{\sqrt{2}}$ then the reference angle is

(a) $\frac{\pi}{3}$ (b) $\frac{-\pi}{4}$ (c) $\frac{-\pi}{3}$ (d) $\frac{\pi}{4}$

53. The solution of the equation $2\sin x + \sqrt{3} = 0$ in 4th quadrant is:

(a) $\frac{\pi}{3}$ (b) $\frac{5\pi}{3}$ (c) $\frac{-\pi}{4}$ (d) $\frac{-\pi}{6}$

ANSWERS TO THE MULTIPLE CHOICE QUESTIONS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
B	B	A	D	A	A	B	B	A	B	B	D	C	C	A	A	B	A
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
B	A	B	B	D	A	A	D	A	B	D	D	D	C	C	C	A	D
37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	
A	A	A	D	B	C	B	C	B	C	C	B	D	B	D	D	B	

SHORT QUESTION'S OF CHAPTER-14 IN ALL PUNJAB BOARDS 2011-2021

1. Solve $1 + \cos x = 0$ $x \in |0, 2\pi|$ **(8 times)**

Sol. $1 + \cos x = 0$

$\cos x = -1$

Since cos x is – ve. There is only one solution $x = \pi$ in $[0,2\pi]$

Since 2π is period of cosx

$\therefore$ General Value of x is $\pi + 2n\pi$, $n \in z$

$S.S = \{\pi + 2n\pi\}$, $n \in z$

2. Find the solution which lies in $\{0 , 2\pi]$ $\cot\theta = \frac{1}{\sqrt{3}}$. **(7 times)**

Sol. $\cot\theta = \frac{1}{\sqrt{3}}$ Taking reciprocal on both sides.

$\Rightarrow$ $\tan\theta = \sqrt{3}$

tan θ is +ve in I and III quadrant, so , the reference angle is $\theta = \frac{\pi}{3}$,

Thus For

I – quadrant.

$\theta = \frac{\pi}{3}$

For III – quadrant.

$\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

Hence the solution is

$\frac{\pi}{3}, \frac{4\pi}{3}$

3. Find the Solution set of the equation: $\sin 4x - \sin 2x = \cos 3x$.

Sol. $\sin 4x - \sin 2x = \cos 3x$

$\because \sin P - \sin\theta = 2\cos\left(\frac{P+\theta}{2}\right)\sin\left(\frac{P-\theta}{2}\right)$

$\Rightarrow$ $2\cos\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) = \cos 3x$

$2\cos\left(\frac{6x}{2}\right)\sin\left(\frac{2x}{2}\right) = \cos 3x$

$2\cos 3x \sin x = \cos 3x$

$2\cos 3x \sin x - \cos 3x = 0$

$\Rightarrow$ $\cos 3x = 0$, $2\sin x - 1 = 0$

$\Rightarrow$ $3x = \frac{\pi}{2}$, $2\sin x = 1$

$\Rightarrow$ $x = \frac{\pi}{6}$, $\sin x = \frac{1}{2}$

$x = (2n+1)\frac{\pi}{6}$

$x = \frac{\pi}{6} + \frac{2n\pi}{6}$

Now, Sin x = $\frac{1}{2}$

As, Sin x is +ve in I and III – quadrant with reference to the angle $\frac{\pi}{6}$.

For I – quadrant: $x = \frac{\pi}{6}$

For II – quadrant: $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

So, $x = \frac{\pi}{6} + 2n\pi$, $\frac{5\pi}{6} + 2n\pi$

There the solution set is.

$= \left\{\frac{\pi}{6} + \frac{2n\pi}{6}\right\} \cup \left\{\frac{\pi}{6} + 2n\pi\right\} \cup \left\{\frac{5\pi}{6} + 2n\pi\right\}$

4. Find the Solution of Secx = -2 in $[0,2\pi]$ **(10 times)**

Sol. Secx = -2

As, $= \frac{1}{Cosx} = -2$

$\Rightarrow$ Cos = $-\frac{1}{2}$

We know that Cosx is –ve in II and III quad.

So, the reference angle is $x = \frac{\pi}{3}$

For II – quadrant:- $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

For III – quadrant:- $x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

Hence solution is $\{\frac{2\pi}{3}, \frac{4\pi}{3}\}$ Ans.

5. Find the value of θ if $2Sin^2\theta - Sin\theta = \theta$. **(6 times 2018)**

Sol. $2Sin^2\theta - Sin\theta = \theta$

$Sin\theta\,(2Sin\theta - 1) = 0$

$\Rightarrow$ $Sin\theta = 0$, $2Sin\theta - 1 = 0$

$\Rightarrow$ $\theta = 0, \pi$, $Sin\theta = \frac{1}{2}$

When $Sin\theta = \frac{1}{2}$

Sinθ is +ve in I and I quadrant with reference angle $\theta = \frac{\pi}{6}$, therefore

For I – quadrant:- $\theta = \frac{\pi}{6}$

For II – quadrant:- $\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Hence solution is

$\{0, \frac{\pi}{6}, \frac{5\pi}{6}\}$ Ans.

6. Solve the equation $sec^2\theta = \frac{4}{3}$. **(2 times)**

Sol. $sec^2\theta = \frac{4}{3}$

$\Rightarrow Cos^2\theta = \frac{3}{4}$

$\Rightarrow Cos\,\theta = \pm\frac{\sqrt{3}}{2}$

If $Cos\,\theta = \frac{\sqrt{3}}{2}$ then $Cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ and Cos θ is +ve in I and IV quadrant

$\therefore \theta = \pi/6, 2\pi - \pi/6$

$= \frac{\pi}{6}, \frac{11\pi}{6}$

If $Cos\,\theta = -\frac{\sqrt{3}}{2}$ then Cos θ is –ve in II and III quadrant

$\therefore \theta = \pi - \frac{\pi}{6}, \quad \pi + \frac{\pi}{6}$

$\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$

Thus

$$\theta = \left\{\frac{\pi}{6} + 2n\pi\right\} \cup \left\{\frac{11\pi}{6} + 2n\pi\right\} \cup \left\{\frac{5\pi}{6} + 2n\pi\right\} \cup \left\{\frac{7\pi}{6} + 2n\pi\right\}$$

Where $n \in Z$

7. **Solve the equation sin2x = cos x.** **(2 times)**

Sol.
$\Rightarrow$ 2Sin x Cos x = Cosx
$\Rightarrow$ 2Sin x Cos x – Cos x = 0
$\Rightarrow$ Cos x (2 Six – 1) = 0
$\therefore$ Cos x = 0 or 2 Sin x – 1 = 0

➤ If Cos x = 0
$\Rightarrow x = \pi/2$ and $x = \frac{3\pi}{2}$
Where $x \in [0, 2\pi]$
$\therefore$ general values of x are
$\frac{\pi}{2} + 2n\pi$ and $\frac{3\pi}{0} + 2n\pi$, $n \in z$

➤ If 2 sin x – 1 = 0 $\Rightarrow$ Sin x = $\frac{1}{2}$

Since Sinx is +ve in I and II quadrants with the reference angle $x = \pi/6$
$\therefore x = \pi/6$ and $x = \pi - \pi/6 = 5\,\pi/6$
Where $x \in [0, 2\pi]$
As 2π is the period of Sin x
$\therefore$ general values of x are
$\pi/6 + 2n\pi$ and $\frac{5\pi}{6} + 2n\pi$, $n \in z$
Hence solution set is
$$= \left\{\frac{\pi}{2} + 2n\pi\right\} \cup \left\{\frac{3\pi}{2} + 2n\pi\right\} \cup \left\{\frac{\pi}{6} + 2n\pi\right\} \cup \left\{\frac{5\pi}{6} + 2n\pi\right\}$$
Where $n \in z$.

8. **Solve the equation $4\cos^2 x - 3 = 0$ in $[0, 2\pi]$** **(2 times)**

Sol. $4\cos^2 x - 3 = 0 \Rightarrow \cos^2 x = 3/4 \Rightarrow \cos x = \pm\frac{\sqrt{3}}{2}$

(i) **if cosx = $\frac{\sqrt{3}}{2}$**

Since cosx is +ive in I and IV quadrant with reference angle $x = \frac{\pi}{6}$
$\therefore x = \frac{\pi}{6}$ and $x = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ where $x \in [0, 2\pi]$
As 2π is the period of cosx
$\therefore$ general values of x are
$\frac{\pi}{6} + 2n\pi$ and $\frac{11\pi}{6} + 2n\pi$, $n \in z$

(ii) **if cos x = $-\frac{\sqrt{3}}{2}$**

Since cosx is "-ve" in II and III quadrant with reference angle $x = \frac{\pi}{6}$
$\therefore x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ and $x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$ where $x \in [0, 2\pi]$
As 2π is the period of cosx
$\therefore$ General values of x are $\frac{5\pi}{6} + 2n\pi$ and $\frac{7\pi}{6} + 2n\pi$, $n \in z$
Hence solution-set is
$$\left\{\frac{\pi}{6} + 2n\pi\right\} u \left\{\frac{11\pi}{6} + 2n\pi\right\} u \left\{\frac{5\pi}{6} + 2n\pi\right\} u \left\{\frac{7\pi}{6} + 2n\pi\right\}, \ n \in z$$

9. **Solve Sinx + Cosx = 0 where x lies in [0, 2π]** **(6 times)**

Sol. Sinx + Cosx = 0

$\frac{\sin x}{\cos x} + \frac{\cos x}{\cos x} =$ (Dividing by cosx $\neq 0$)

$\Rightarrow$ tanx + 1 = 0 $\Rightarrow$ tanx = -1

$\because$ tanx is –ive in II and IV quadrants with reference angle $x = \pi/_4$

$\therefore x = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ where $x \in [0, \pi]$

As π is the period of tanx. $\therefore$ general value of x is $\frac{3\pi}{4} + n\pi$, n∈z

Solution set = $\left\{\frac{3\pi}{4} + n\pi\right\}$, n∈z

10. Solve the trigonometric equation $\tan^2\theta = 1/_3$; $\theta \in [0.2\pi]$. (5 times)

Sol. $\tan^2\theta = \frac{1}{3}$

$\Rightarrow \tan\theta = \pm\frac{1}{\sqrt{3}}$

If $\tan\theta = \frac{1}{\sqrt{3}}$ and tanθ is +ive in I and III quadrant.

$\Rightarrow \theta = \frac{\pi}{6}$, $\theta = \pi + \frac{\pi}{6}$

$\theta = \frac{\pi}{6}$ $\theta = \frac{7\pi}{6}$

If $\tan\theta = -\frac{1}{\sqrt{3}}$ $\because \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$ and tanθ is –ive in II and VI quadrant.

$\Rightarrow \theta = \pi - \frac{\pi}{6}$ $2\pi - \frac{\pi}{6}$

$= \frac{5\pi}{6}$, $\frac{11\pi}{6}$

Thus

$\theta = \left\{\frac{\pi}{6} + n\pi\right\} U \left\{\frac{7\pi}{6} + n\pi\right\} U \left\{\frac{5\pi}{6} + n\pi\right\} U \left\{\frac{11\pi}{6} + n\pi\right\} n \in z$

11. Solve $Cosec^2\theta = \frac{4}{3}$; $\theta \in |0, 2\pi|$ **(2 times 2018)**

Sol. $Cosec^2\,\theta = 4/_3$

$\Rightarrow$ $Sin^2\,\theta = 3/_4$

$Sin\,\theta = \pm\frac{\sqrt{3}}{2}$

$Sin\,\theta = \frac{\sqrt{3}}{2}$

$\because Sin\,\pi/_3 = \sqrt{3}/_2$ and Sin θ is + ve

In I and II quadrant

$\because \theta = \pi/_3$; $\pi - \pi/_3 = 2\,\pi/_3$

$\theta = \pi/_3$, $2\pi/_3$

If $Sin\,\theta = -\frac{\sqrt{3}}{2}$

Sin θ is – ve in III and IV quad.

$\therefore \theta = \pi + \frac{\pi}{3}$, $2\pi - \frac{\pi}{3}$

$= \frac{4\pi}{3}$, $5\pi/_3$

Thus $\theta = \left\{\frac{\pi}{3} + 2n\pi\right\} \cup \left\{\frac{2\pi}{3} + 2n\pi\right\} \cup \left\{\frac{4\pi}{3} + 2n\pi\right\} \cup \left\{\frac{5\pi}{3} + 2n\pi\right\}$ n ∈ z

12. Solve the Equation $Cosec^2\theta = \frac{4}{3}$ **(2 times)**

Sol. $Sin^2\,\theta = \frac{3}{4}$

$Sin\,\theta = \pm\frac{\sqrt{3}}{2}$

$Sin\,\theta = \frac{\sqrt{3}}{2}$ $Sin\,\theta = -\frac{\sqrt{3}}{2}$

$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ $\theta = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\theta = 60^\circ$ $\theta = -60^\circ$

Sin θ is +ve Sin θ is –ve

In I, II Quad In II, IV Quad

$\therefore \theta = $ I, II $\theta = $ III, IV

$\theta = , 180^\circ - \theta$ $\theta = 180^\circ + \theta , 360^\circ - \theta$

$\theta = 60^\circ , 180^\circ - 60^\circ$ $\theta = 180^\circ + \theta , 360^\circ - 60^\circ$

$\theta = 60^\circ , 120^\circ$ $\theta = 240^\circ , 300^\circ$

$\theta = \frac{60\pi}{180} , \frac{120\pi}{180}$ $\theta = \frac{240\pi}{180} , \frac{300\pi}{180}$

$\theta = \frac{\pi}{3} , \frac{2\pi}{3}$ $\theta = \frac{4\pi}{3} , \frac{5\pi}{3}$

13. Find the solution of Cosec $\theta = 2$ which the $(0, 2\pi)$ **(6 times)**

Sol. Cosec $\theta = 2$

$\sin\theta = \frac{1}{2}$

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$

$\theta = 30^\circ$

$\theta = $ I, II

$\theta = 30^\circ , 180^\circ - \theta$

$\theta = 30^\circ , 180^\circ - 30^\circ$

$\theta = 30^\circ , 150^\circ$

$\theta = \frac{30\pi}{180} , \frac{150\pi}{180}$

$\theta = \frac{\pi}{6} , \frac{5\pi}{6}$

14. Find the solution of Cot $\theta = \frac{1}{\sqrt{3}}$ where $\theta \in (0, 2\pi)$. **(4 times)**

Sol. Cot $\theta = \frac{1}{\sqrt{3}}$

$Tan\,\theta = \sqrt{3}$

$\theta = \tan^{-1}(\sqrt{3})$

$\theta = 60^\circ$

$\theta = $ I, III (Tan θ is +ve)

$\theta = 60^\circ , 180^\circ + 60^\circ$

$\theta = 60^\circ , 240^\circ$

$\theta = \frac{60\pi}{180} , \frac{240\pi}{180}$

$\theta = \frac{\pi}{3} , \frac{4\pi}{3}$

15. Find the solution of Cot $\theta = \frac{-1}{\sqrt{3}}$ **(2 times)**

Sol. Cot $\theta = \frac{-1}{\sqrt{3}}$

Tan $\theta = -\sqrt{3}$

$\theta = \tan^{-1}(-\sqrt{3})$

$\theta = -60^\circ$

$\theta = $ II, IV $(Tan\theta - ve)$

$\theta = 180^\circ - 60^\circ , 360^\circ - 60^\circ$

$\theta = 120^\circ , 300^\circ$

$\theta = 120^\circ , 300^\circ$

$\theta = \frac{120\pi}{180} , \frac{300\pi}{180}$

$\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$

16. Solve $2\text{Sin}\,\theta + \text{Cos}^2\theta - 1 = 0$, $\theta \in \{0, 2\pi\}$ (5 times)

Sol. $2\text{Son}\,\theta + \text{Cos}^2\theta - 1 = 0$

$2\,\text{Sin}\,\theta - 1 + \text{Cos}^2\theta = 0$

$2\text{Sin}\,\theta - (1 - \text{Cos}^2) = 0$

$2\text{Sin}\,\theta - \text{Sin}^2\theta = 0$

$\text{Sin}\,\theta\,(2 - \text{Sin}\,\theta) = 0$

$\text{Sin}\,\theta = 0$, $2 - \text{Sin}\,\theta = 0$

$\theta = \text{Sin}^{-1}(0)$ $\text{Sin}\,\theta = 2 \notin (-1, 1)$

Here sol does not Exist

$\therefore \theta = \text{Sin}^{-1}(0)$

$\theta = 0°, 180°$

$\theta = 0, \pi$

17. Solve $\text{SinxCos}\,x = \frac{\sqrt{3}}{4}$ $x \in (0, \pi)$ (3 times)

Sol. $\text{Sin}\,x\,\text{Cos}\,x = \frac{\sqrt{3}}{4}$

$2\text{Sinx}\,\text{Cos}\,x = 2\left(\frac{\sqrt{3}}{4}\right)$

$\text{Sin}2x = \frac{\sqrt{3}}{2}$

$2x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$2x = 60°$

$2x = \text{I}, \text{II}$ (Sin θ is +ve)

$2x = 60°, 180° - 60°$

$2x = 60°, 120°$

$2x = \frac{60\pi}{180}, \frac{120\pi}{180}$

$2x = \frac{\pi}{3}, \frac{2\pi}{3}$

$x = \frac{\pi}{6}, \frac{2\pi}{6}$

18. Solve the equation tax $x = -1$ in $[0, 2\pi]$

Sol Given $\tan x = -1$

$x = \tan^{-1}(-1)$

$as \quad \tan^{-1}(1)$

$x = \frac{\pi}{4}$

Tanx is -ve in II and IV Quad with the reference angle $x = \frac{\pi}{4}$

II Quad

$x = \pi - \theta$

$x = \pi - \frac{\pi}{4}$

$x = \frac{4\pi - \pi}{4}$

$x = \frac{3\pi}{4}$

IV Quad

$x = 2\pi - \theta$

$x = 2\pi - \frac{\pi}{4}$

$x = \frac{8\pi - \pi}{4}$

$x = \frac{7\pi}{4}$

Sol set: $\frac{3\pi}{4}, \frac{7\pi}{4}$ with $n \in Z$

19. Solve the equation $\text{Sec}\theta = -\frac{2}{\sqrt{3}}$ in $[0, 2\pi]$

Sol $Sec\theta = -\frac{2}{\sqrt{3}}$

$\frac{1}{Cos\theta} = \frac{-2}{\sqrt{3}}$

$Cos\theta = \frac{-\sqrt{3}}{2}$

$\theta = \frac{\pi}{6}$

cos θ is -ve in II and III Quad with the reference angle $\frac{\pi}{6}$

II Quad

$\theta = \pi - \frac{\pi}{6}$

$\theta = \frac{6\pi - \pi}{6}$

$\theta = \frac{5\pi}{6}$

III Quad

$\theta = \pi + \frac{\pi}{6}$

$\theta = \frac{6\pi + \pi}{6}$

$\theta = \frac{7\pi}{6}$

Sol Set: $\left\{\frac{5\pi}{6}, \frac{7\pi}{6}\right\}$ and $n \in z$

20. Define trigonometric equation.

Sol **TrignometricEquation:-**

An equation, containing at least one trigonometric function is called trigonometric equation.

For Example: $\text{Sinx} = \frac{2}{5}$, secx = tanx

2018

21. Show that $Sinx = -\frac{\sqrt{3}}{2}$ **lies in [0 , 2π]** **(2 times)**

Sol: Given that $Sinx = -\frac{\sqrt{3}}{2}$

Sin is –ve in III and IV equal with respect to angle $-\frac{\sqrt{3}}{2}$

$x = \pi + \frac{\pi}{3}$ and $x = 2\pi - \frac{\pi}{3}$ $\therefore x \in [0, 2\pi]$

⇨ $x = \frac{4\pi}{3}$, $x = \frac{5\pi}{3}$

$\therefore$ $S.S = \left\{\frac{4\pi}{3}, \frac{5\pi}{3}\right\}$ Ans.

2019

22: Solve $Sinx = \frac{1}{2}$ **(4 times)**

Sol: Given $Sinx = \frac{1}{2}$ $x = Sin^{-1}\left(\frac{1}{2}\right)$

$x = \frac{\pi}{6}$ $\because Sin\frac{\pi}{6} = \frac{1}{2}$

Since $Sinx$ is positive in I and II Quadrant with reference angle is $\frac{\pi}{6}$

So

I Quad II Quad

$x=\frac{\pi}{6}$ $=\pi-\frac{\pi}{6}$

$=\frac{6\pi-\pi}{6}$ $x=\frac{5\pi}{6}$

Hence $S.S\ in\ [0,2\pi]=\left\{\frac{\pi}{6},\frac{5\pi}{6}\right\}$ with $n\in Z$

23: Solve the trigonometric equation $\tan\theta=\frac{1}{\sqrt{3}}$

Sol: Given $\tan\theta=\frac{1}{\sqrt{3}}$

$\theta=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Since $\tan\theta$ is positive in I & III Quadrant with reference angle is $\frac{\pi}{6}$

So

I Quad

$\theta=\theta$

$\theta=\frac{\pi}{6}$

III Quad

$\theta=\pi+\theta$

$\theta=\pi+\frac{\pi}{6}$

$\theta=\frac{6\pi+\pi}{6}$

$\theta=\frac{7\pi}{6}$

Hence

$S.S\ in\ [0,2\pi]=\left\{\frac{\pi}{6},\frac{7\pi}{6}\right\}$ with $n\in Z$

24: Solve the equation $Sin^2x+Cosx=1$

Sol: $Sin^2x+Cosx=1$ $\because Sin^2x=1-Cos^2x$

$1-Cos^2x+Cosx-1=0$

$\Rightarrow -Cos^2x+Cosx=0$

$\Rightarrow -Cosx(Cosx-1)=0$

Either $-Cosx=0$ Or $Cosx-1=0$

$\Rightarrow Cosx=0$, $Cosx=1$

$\Rightarrow x=Cosx^{-1}(0)$, $x=Cosx^{-1}(1)$

$x=\frac{\pi}{2},\frac{3\pi}{2}$, $x=0,2\pi$ where $x\in[0,2\pi]$

Hence $S.S=\left\{\frac{\pi}{2}+2n\pi\right\}\cup\left\{\frac{3\pi}{2}+2n\pi\right\}\cup\{2n\pi\}$, with $n\in Z$

Board Papers 2021

Sahiwal Board Group-1

(Intermediate Part – I, Class 11th)

Objective

Mathematics
Session (2021)

Time : 30 Minutes
Marks : 20

Note: You have four choices for each objective type question as A, B, C and D. the choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

1. Rank of matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is:
(a) 1 (b) 2 (c) 3 (d) 4

2. If a, A, b are in A.P, then 2A is:
(a) a –b (b) $\frac{a+b}{2}$ (c) a + b (d) b – a

3. If $a_{n-3} = 2n - 5$ its nth term is:
(a) 2n + 1 (b) 2n + 3 (c) 2n – 2 (d) 2n – 8

4. $\frac{x^2+1}{(x-1)(x+2)}$ is
(a) proper fraction (b) improper fraction (c) identity (d) both B&C

5. Radius of the inscribed circle is:
(a) $r = \frac{\Delta}{s}$ (b) $r = \frac{abc}{4\Delta}$ (c) $r = \frac{s}{\Delta}$ (d) $r = \frac{s-a}{\Delta}$

6. An equation of the form $ax^2 + bx + c = 0$ is called quadratic if:
(a) a = 0 (b) b = 0 (c) $a \neq 0$ (d) $b \neq 0$

7. Let A,G,H b the A.M, G.M and H.M between a and b respectively, then G^2=.
(a) A + H (b) $\sqrt{ab}$ (c) $\frac{A}{H}$ (d) AH

8. $\sin\left(\frac{3\pi}{2} - \theta\right)$
(a) $\sin\theta$ (b) $\cos\theta$ (c) $-\cos\theta$ (d) $-\sin\theta$

9. $\{x | x \in E \wedge 4 < x < 6\}$ equals:
(a) {4} (b) {5} (c) {6} (d) ϕ

10. Which angle is quadrantal angle?
(a) 120° (b) 270° (c) 60° (d) 45°

11. If $\sin x = \frac{-\sqrt{3}}{2}$, then solution is:
(a) $\frac{4\pi}{6}, \frac{5\pi}{6}$ (b) $\frac{4\pi}{3}, \frac{5\pi}{3}$ (c) $\frac{5\pi}{6}, \frac{7\pi}{6}$ (d) $\frac{\pi}{3}, \frac{7\pi}{3}$

12. $Tan^{-1}(-\sqrt{3}) =$
(a) $\frac{2\pi}{3}$ (b) $\frac{-2\pi}{3}$ (c) $\frac{-\pi}{6}$ (d) $\frac{-\pi}{3}$

13. If one root of the equation $x^2 - 3x + a = 0$ is 2, than a is:
(a) 2 (b) -2 (c) 3 (d) -3

14. If a,b,c have their usual meanings then $\frac{c^2+a^2-b^2}{2ac}$ =
(a) $\cos\alpha$ (b) $\cos\beta$ (c) $\cos\gamma$ (d) $\sin\beta$

15. The period of 3sin3x is:
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) π

16. If A is a matrix of order 3 x 4 , then the order of AA^t is:
(a) 4×3 (b) 3×3 (c) 3×4 (d) 4×4

17. If $\cos\theta=\frac{1}{\sqrt{2}}$, then θ is equal to:
(a) 30° (b) 45° (c) 60° (d) 90°

18. Any real number "a" is equal to:
(a) ia (b) (a,b) (c) a (d) (b,a)

19. If n is any positive integer, then $2^n > 2(n+1)$ is true for all.
(a) $n\le3$ (b) $n<3$ (c) $n\ge3$ (d) $n>3$

20. $\frac{^nP_r}{r!}$ is equal to:
(a) ^nC_r (b) $^nC_{r-1}$ (c) $^{n+1}C_r$ (d) $^{n-1}C_r$

Sahiwal Board Group-I

MATHEMATICS PAPER-II **TIME ALLOWED : 2.30 Hours**
GROUP-II **SUBJECTIVE** **MAXIMUM MARKS:80**

NOTE: Write same question number and its part number on answer book, as given in the question paper.

SECTION-I

2. Attempt any eight parts.

(i) Find the multiplicative inverse of $\left(\sqrt{2},-\sqrt{5}\right)$

(ii) Prove that $-\frac{7}{12}-\frac{5}{18}=\frac{-21-10}{36}$

(iii) If $z_1 = 2 + i$, $z_2 = 3 - 2i$, $z_3 = 1 + 3i$ then express $\frac{\overline{z_1 z_3}}{z_2}$ in the form a+ ib

(iv) Write the inverse and contrapositive of conditional $\sim p\rightarrow\sim q$

(v) Show that A – B and B - A by Venn diagram when A and B are overlapping sets.

(vi) If a and h are elements of a group G, then solve the equation xa = b

(vii) Find the matrix X if $X\begin{bmatrix}5 & 2\\ -2 & 1\end{bmatrix}=\begin{bmatrix}-1 & 5\\ 12 & 3\end{bmatrix}$

(viii) Show that $\begin{vmatrix}1 & 1 & 1\\ x & y & z\\ yz & zx & xy\end{vmatrix}$

(ix) If $A=\begin{bmatrix}-1 & 2\\ 1 & 4\\ 2 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 3\\ -2 & 1\end{bmatrix}$ show that $(AB)^t = B^tA^t$

(x) Find two consecutive numbers, whose product is 132.

(xi) Evaluate $\left(-1+\sqrt{-3}\right)^5+\left(-1-\sqrt{-3}\right)^5$

(xii) Find numerical value of K, if the polynomial $x^2 + kx^2 - 7x + 6$ has a remainder of -4, when divided by x + 2.

3. Attempt any eight parts.

(i) Write into partial fraction form of $\frac{4x^2}{\left(x^2+1\right)^2\left(x-1\right)}$ without finding constants.

(ii) Write into partial fraction form of $\frac{1}{\left(x-1\right)^2\left(x^2+2\right)}$ without finding constants.

(iii) 1f $a_{n-3} = 3n-11$, find nth term of the sequence.

(iv) Find the Geometric Mean between -2i and 8i.

(v) If $y = 1 + 2x + 4x^2 + 8x^3 + \ldots$ Show that $x = \dfrac{y-1}{2y}$

(vi) Find 8^{th} term of H.P; $\dfrac{1}{2}, \dfrac{1}{5}, \dfrac{1}{8}, \ldots$

(vii) Write $\dfrac{52.51.50.49}{4.3.2.1}$ in the factorial form.

(viii) Find the value of n when ${}^{11}P_n = 11.10.9$

(ix) Find the value of n when ${}^{n}C_5 = {}^{n}C_4$

(x) Show that the inequality $4^n > 3^n + 4$ is true, for integral values of $n \geq 2$.

(xi) Calculate $(9.98)^4$ by means of binomial theorem

(xii) Expand $(1+x)^{\frac{-1}{3}}$ upto 4 terms.

4. Attempt any nine parts.

(i) Convert 54° 45° into radians. (ii) Verify that $2\sin 45^o + \dfrac{1}{2}\operatorname{cosec} 45^o = \dfrac{3}{\sqrt{2}}$

(iii) Prove that $\sec^2\theta - \operatorname{cosec}^2\theta = \tan^2\theta - \cot^2\theta$

(iv) Prove that $\sin(180^o + \alpha)\sin(90^o - \alpha) = -\sin\alpha\cos\alpha$

(v) Prove that cot $\cot\alpha - \tan\alpha = 2\cot 2\alpha$

(vi) Express cos12° + cos 48° as product.

(vii) Find the period of $\sin\dfrac{x}{3}$ (viii) Find θ if $\sin\theta = 0.5791$

(ix) Find the smallest angle of the triangle ABC. when a =37.34. b =3.24. c=35.06

(x) Find the value of R if a = 13, b = 14 , c = 15

(xi) Find the value of $\cos\left(\sin^{-1}\dfrac{1}{\sqrt{2}}\right)$ (xii) Solve the equation $1 + \cos x = 0$

(xiii) Find the solution of sec x = -2

SECTION-II

5.(a) Find the rank of the matrix $\begin{bmatrix} 1 & -4 & -7 \\ 2 & -5 & 1 \\ 1 & -2 & 3 \\ 3 & -7 & 4 \end{bmatrix}$

(b) Use synthetic division to find the values of a and b if -2 and 2 are the roots of the polynomial $x^3 - 4x^2 + ax + b$

6.(a) Resolve into partial fraction $\dfrac{x^2}{(x-2)(x-1)^2}$

(b) The sum of an finite geometric series is 9 and the sum of squares of its terms is $\dfrac{81}{5}$ Find the series.

7.(a) Two dice are thrown. What is the probability that the sum of the number of dots appearing on them is 4 or 6?

(b) Find the coefficient of x^5 in the expansion of $\left(x^2 - \dfrac{3}{2x}\right)^{10}$

8.(a) Find the values of the trigonometric functions of the angle $\dfrac{-17}{3}\pi$

(b) Prove that $\dfrac{\cos 3\theta}{\cos\theta} + \dfrac{\sin 3\theta}{\sin\theta} = 4\cos 2\theta$

9.(a) Prove that $\dfrac{1}{r^2} + \dfrac{1}{r_1^2} + \dfrac{1}{r_2^2} + \dfrac{1}{r_3^2} = \dfrac{a^2 + b^2 + c^2}{\Delta^2}$ (b) Prove that $\tan^{-1}\dfrac{120}{119} = 2\cos^{-1}\dfrac{12}{13}$

Multan Board Group-I

Mathematics (Intermediate Part – I, Class 11th) Time : 30 Minutes

Session (2021) **Objective** Marks : 20

Note: You have four choices for each objective type question as A, B, C and D. the choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

1. If A and B are non empty disjoint sets then:
(a) $A\cap B=A$ (b) $A\cap B=B$ (c) $A\cap B=\phi$ (d) $A\cap B\neq\phi$

2. If z = -2 + 3i then $\bar{z}$ =
(a) -2 – 3i (b) 2 – 3i (c) - 2 + 3i (d) 2 + 3i

3. If $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then Adj A =
(a) $\begin{bmatrix} -a & -b \\ c & d \end{bmatrix}$ (b) $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
(c) $\begin{bmatrix} a & -b \\ -c & d \end{bmatrix}$ (d) $\begin{bmatrix} -a & -b \\ c & -d \end{bmatrix}$

4. If $|A|=5$ then $|A'|=$
(a) 1/5 (b) 0 (c) -5 (d) 5

5. Sum of all the four fourth roots of unity is:
(a) 0 (b) 1 (c) -1 (d) 4

6. The product of roots of the equation $3x^2 + 5x = 0$
(a) $\frac{-5}{3}$ (b) $\frac{5}{3}$ (c) 5 (d) 0

7. An equation which is true for all values of unkown is called:
(a) Identity (b) Algebric equation (c) Alegbric relation (d) Contional equation

8. The A.M between $1 - x + x^2$ and $1 + x + x^2$ is
(a) x + 1 (b) $x^2 + 1$ (c) $\frac{x+1}{2}$ (d) $\frac{x^2+1}{2}$

9. G.M between 2 and 8 is/are:
(a) 5 (b) 8 (c) ± 4 (d) 16

10. The sum of an infinite geometric series with $|r|<1$, where first term is a and r is common ratio:
(a) $\frac{a}{1+r}$ (b) $\frac{a}{1-r^2}$ (c) $\frac{a}{1-r}$ (d) $\frac{a}{1+r^2}$

11. If $^nP_2 = 30$, then n =
(a) 6 (b) 4 (c) 5 (d) 8

12. General term in the expansion of $(a + x)^n$ is:
(a) $\binom{n}{r}a^{n-r}x^r$ (b) $\binom{n}{r}a^r x^n$ (c) $\binom{n}{r}a^n x^{n-r}$ (d) $\binom{n}{r}a^n x^n$

13. $\frac{5\pi}{4}$ radian =
(a) 360° (b) 225° (c) 335° (d) 270°

14. $(\cos 2\theta)^2+(\sin 2\theta)^2=$
(a) 0 (b) 2 (c) 4 (d) 1

15. $\sin(180^\circ+\alpha)=$
(a) $-\cos\alpha$ (b) $\sin\alpha$ (c) $\cos\alpha$ (d) $-\sin\alpha$

16. Period of $\tan\frac{x}{3}$ is:
(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) 3π

17. In any triangle ABC, with usual notation, r_1 =

(a) $\frac{\Delta}{s-a}$ (b) $\frac{\Delta}{s-b}$ (c) $\frac{\Delta}{s-c}$ (d) $\frac{\Delta}{s}$

18. Circum radius R =

(a) $\frac{\Delta}{abc}$ (b) $\frac{\Delta}{s}$ (c) $\frac{abc}{4\Delta}$ (d) $\frac{\Delta}{s-a}$

19. $\cos\left(\sin^{-1}\frac{1}{\sqrt{2}}\right)=$

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{\pi}{4}$ (d) $-\frac{\pi}{4}$

20. If $\sin x=\frac{\sqrt{3}}{2}\, and\, x\in[0,2\pi]\, then\, x=$

(a) $\frac{5\pi}{3},\frac{4\pi}{3}$ (b) $\frac{\pi}{4},\frac{3\pi}{4}$ (c) $\frac{\pi}{3},\frac{2\pi}{3}$ (d) $\frac{\pi}{6},\frac{5\pi}{6}$

Multan Board Group-I

MATHEMATICS PAPER-II **TIME ALLOWED : 2.30 Hours**
GROUP-II **SUBJECTIVE** **MAXIMUM MARKS:80**

NOTE: Write same question number and its part number on answer book, as given in the question paper.

SECTION-I

2. Attempt any eight parts.

(i) Find the modulus of the complex number $1-i\sqrt{3}$
(ii) Simplify (2,6) ÷ (3, 7)
(iii) Name the property used in the following equation a(b - c) = ab - ac
(iv) Write two proper subsets of the set {a, b, c }
(v) Construct the truth table of the following statement $(p\wedge\sim p)\rightarrow q$
(vi) Find the solution of the linear equation xa = b. where a and b belong to group G.
(vii) Find x and y if $\begin{bmatrix}2 & 0 & x\\ 1 & y & 3\end{bmatrix}+2\begin{bmatrix}1 & x & y\\ 0 & 2 & -1\end{bmatrix}=\begin{bmatrix}4 & -2 & 3\\ 1 & 6 & 1\end{bmatrix}$
(viii) Without expansion verify $\begin{vmatrix}1 & a^2 & \frac{a}{bc}\\ 1 & b^2 & \frac{b}{ca}\\ 1 & c^2 & \frac{c}{ab}\end{vmatrix}=0$
(ix) Solve the equation by using the quadratic formula $16x^2 + 8x + 1 = 0$
(x) Evaluate $(1 + w - w^2)(1 - w + w^2)$
(xi) Find the inverse of matrix $\begin{bmatrix}-2 & 3\\ -4 & 5\end{bmatrix}$
(xii) If α,β are roots of the equation $x^2 - px - p - c = 0$ then prove that $(1+\alpha)(1+\beta)=1-c$

3. Attempt any eight parts.

(i) Resolve $\frac{1}{(x+1)^2(x^2-1)}$ into partial fractions without finding the constants.
(ii) Resolve $\frac{4x^2}{(x^2+1)^2(x-1)}$ into partial fractions without finding constants.
(iii) If $a_{n-3}=2n-5$ find the nth term of the sequence.
(iv) Find A.M. between x - 3 and x + 5

(v) If 5 is harmonic mean between 2 and b, Find b

(vi) Find the 12^{th} term of the harmonic sequence 1/3 , 2/9, , 1/6

(vii) Find the value of n if ${}^{n}P_4 : {}^{n-1}P_3 = 9 : 1$

(viii) How many necklaces can be made by 6 beads of different colours?

(ix) How many diagonals can be made by 8 sided figure?

(x) Verify the statement $2 + 6 + 18 + ... + 2 \times 3^{n-1} = 3^n - 1$ for n = 1 , 2

(xi) Expand $(4 - 3x)^{1/2}$ upto 3 terms.

(xii) If x be so small that its square and higher powers be neglected, prove that $\frac{1-x}{\sqrt{1+x}} \approx 1 - \frac{3}{2}x$

4. Attempt any nine parts.

(i) Find r, when $l = 5cm$, $\theta = \frac{1}{2}$ radian.

(ii) Write any two fundamental identities of trigonometry.

(iii) Evaluate $\frac{1 - \tan^2 \pi/3}{1 + \tan^2 \pi/3}$

(iv) If α, β, γ are angles of triangle ABC then prove that $\cos(\alpha + \beta) = -\cos\gamma$

(v) Prove that $\tan(45^o + A)\tan(45^o - A) = 1$

(vi) Prove that $\frac{\sin 8x + \sin 2x}{\cos 8x + \cos 2x} = \tan 5x$

(vii) Find the period of $\sin \frac{x}{5}$

(viii) A kite is flying at a height of 67.2m is attached to a fully stretched string inclined at an angle of 55° to the horizontal. Find the length of string.

(ix) Find the area of triangle ABC, when b = 37, c = 45, $\alpha = 30° 50'$

(x) Prove that $r r_1 r_2 r_3 = \Delta^2$

(xi) Show that $\tan(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}$

(xii) Solve the equation $\sin x = \frac{1}{2}$, where $x \in [0, 2\pi]$

(xiii) Find solution of the equation sec x = -2 which lies in the interval $[0, 2\pi]$

SECTION-II

5.(a) If $A = \begin{bmatrix} -1 & 2 \\ 1 & 4 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$ then verify $(AB)' = B'A'$

(b) If 'w' is a root of $x^2 + x + 1 = 0$ show that its other root is w^2 and prove that $w^3 = 1$

6.(a) Resolve $\frac{x^2+1}{x^3+1}$ into partial fractions.

(b) Find four Arithmetic Means (A.Ms) between $\sqrt{2}$ and $\frac{12}{\sqrt{2}}$

7.(a) Find the values of n and r, when ${}^{n}C_r = 35$ and ${}^{n}P_r = 210$

(b) Find the coefficient of x^5 in the expansion of $\left(x^2 - \frac{3}{2x}\right)^{10}$

8.(a) Prove that $\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

(b) Prove that $\frac{2\sin\theta\sin(2\theta)}{\cos\theta + \cos(3\theta)} = \tan(2\theta)\tan\theta$

9.(a) The sides of a triangle are $x^2 + x + 1$, $2x + 1$ and $x^2 - 1$. Prove that the greatest angle of the triangle is 120°.

(b) Prove that $\tan^{-1}\frac{1}{11} + \tan^{-1}\frac{5}{6} = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$

D.G.Khan Board Group-I

Mathematics (Intermediate Part – I, Class 11th) Time : 30 Minutes
Session (2021) **Objective** Marks : 20

Note: You have four choices for each objective type question as A, B, C and D. the choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

1. Multiplicative identity in complex number is
(a) (0,0) (b) (0,1) (c) (1,1) (d) (1,0)

2. Set $\{1, w, w^2\}$ is closed w.r.t
(a) Addition (+) (b) Multiplication (×)
(c) Both A and B (d) Division (÷)

3. Let A be a square matrix than $|A'| =$
(a) A^{-1} (b) $|A'|$ (c) $|A|$ (d) Not defined

4. If A is a matrix of order 3×1, then the order of AA^t is
(a) 1×3 (b) 1×1 (c) 3×3 (d) 3×1

5. If $x^{1/4} = -2$ then x =
(a) 8 (b) -8 (c) 16 (d) -16

6. Remainder is 11 if $x^2 + 3x + 7$ is divided by
(a) x + 1 (b) x + 2 (c) x + 3 (d) x – 1

7. The number of co-efficients in the partial fraction of $\frac{1}{(x-1)^2(x^2+16)}$ are
(a) 2 (b) 3 (c) 4 (d) 5

8. 26th term of an = $(-1)^{n+1}$
(a) 1 (b) -1 (c) 26 (d) -26

9. Relation between A, G, H is
(a) A > G > H (b) A < G < H (c) Both A and B (d) A > G < H

10. Reciprocal of the sequence 1/3, 1/5, ... forms
(a) Geomatric sequence (b) Arithmatic sequence
(c) Harmonic sequence (d) Null sequence

11. ${}^{n+1}C_r + {}^{n+1}C_{r-1} = 0$
(a) ${}^{n+1}C_r$ (b) ${}^{n+2}C_{r-1}$ (c) ${}^{n+1}C_{r+1}$ (d) ${}^{n+2}C_r$

12. In the middle term T_{r+1} of the binomial expansion of $(a+b)^{12}$, γ =
(a) 6 (b) 7 (c) 5 (d) 12

13. Which of the following is quadrental Angle
(a) 350° (b) -390° (c) -360° (d) 410°

14. $\frac{-9\pi}{2}$ coincides with
(a) OX (b) OY (c) OX' (d) OY'

15. Sin (-300°)=
(a) $\frac{-\sqrt{3}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{2}}$

16. The period of $3\sin\frac{x}{3}$ is:
(a) 6π (b) 2π (c) 3π (d) 4π

17. The radius of inscribed circle is:
(a) $\frac{abc}{4\Delta}$ (b) $\frac{\Delta}{s}$ (c) $\frac{\Delta}{s-a}$ (d) $\frac{\Delta}{s-b}$

18. $\frac{c^2\sin\alpha\sin\beta}{\sin\gamma} =$
(a) Δ (b) $\frac{\Delta}{2}$ (c) 2Δ (d) ΔS

19. $\cos(\tan^{-1}(0))$

(a) 0 (b) -1 (c) 1 (d) ∞

20. If $\cos x = 0$ the number of solutions are

(a) 2 (b) 4 (c) 6 (d) infinite

D.G.KHan Board Group-I

MATHEMATICS PAPER-II **TIME ALLOWED : 2.30 Hours**

GROUP-II **SUBJECTIVE** **MAXIMUM MARKS:80**

NOTE: Write same question number and its part number on answer book, as given in the question paper.

SECTION-I

2. Attempt any eight parts.

(i) Check the closure property in the set {0 , -1 } w .r.t addition and multiplication

(ii) Find the multiplicative inverse of the number $\left(\sqrt{2}, -\sqrt{5}\right)$

(iii) If Z is any complex number , then prove that $Z\overline{Z} = |Z|^2$

(iv) Write the descriptive form and tabular form of the set $\{x | x \in O \wedge 5 \leq x \leq 7\}$

(v) Show that the statement $(p \wedge q) \rightarrow P$ is a tautology

(vi) Show that the set of natural number N is non-commutative and non-associative w.r.t subtraction.

(vii) Find the values of x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$

(viii) Find the matrix X, if $X\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$

(ix) Find the value of λ if matrix $A = \begin{bmatrix} 4 & \lambda & 3 \\ 7 & 3 & 6 \\ 2 & 3 & 1 \end{bmatrix}$ is singular

(x) Find the roots of the equation $5x^2 - 13x + 6 = 0$

(xi) Find four fourth roots of unity

(xii) When the polynomial $x^4 + 2x^3 + kx^2 + 3$ is divided by x -2 , the remainder is 1. Find the value of k.

3. Attempt any eight parts.

(i) Resolve $\dfrac{9}{(x+2)^2(x-1)}$ into partial fraction without finding the constants A,B and C

(ii) Resolve $\dfrac{3x+7}{(x^2+4)(x+3)}$ into partial fraction without finding the constants A,B and C

(iii) Which term of the A.P -2 , 4, 10, ... is 148?

(iv) Find the 5th term of the G.P 3 , 6, 12,...

(v) Find the sum of the infinite G.P $2, \sqrt{2}, 1, \ldots$

(vi) Find A, G, H if $a = \dfrac{-2}{5}, b = \dfrac{-8}{5}$

(vii) Evaluate ${}^9P_8$

(viii) How many arrangements of the letters of the word ATTACKED" can be made if each arrangement begins with C and ends with K .'

(ix) Find the value of n when ${}^nC_{12} = {}^nC_6$

(x) Show that the inequality $4^n > 3^n + 4$ is true for n = 2 , 3

(xi) Calculate $(9.98)^4$ by using binomial theorem.

(xii) Expand $(8-2x)^{-1}$ up to 4 terms by using binomial theorem.

4. Attempt any nine parts.

(i) Express the sexagesimal measure of angle 120'40" in radian

(ii) Verify $\sin 2\theta = 2\sin\theta\cos\theta$, when $\theta = 30°, 45°$

(iii) Prove that $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$, where θ is not and odd multiple of $\frac{\pi}{2}$

(iv) Without using the tables, Find the value of cot (- 855°)

(v) Prove that $\dfrac{1-\tan\theta\tan\phi}{1+\tan\theta\tan\phi} = \dfrac{\cos(\theta+\phi)}{\cos(\theta-\phi)}$

(vi) Express the difference $\sin 8\theta - \sin 4\theta$ as product

(vii) Find the period of $3\cos\frac{x}{5}$

(viii) A vertical pole is 8m high and length of its shadow is 6m .What is the angle of elevation of the sun at that moment?

(ix) Find the smallest angle of the triangle ABC . when a = 37.34 , b = 3.24 , c = 35.06

(x) Find the area of a triangle ABC, when b =37,c =45 , $\alpha = 30°50'$

(xi) Without using tables/calculator , Find $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(xii) Find the solution of $\sin x = \frac{-\sqrt{3}}{2}$ which lie in $[0, 2\pi]$

(xiii) Solve the trigonometric equation $\tan^2\theta = \frac{1}{3}$ in $[0, 2\pi]$

SECTION-II

Attempt any three.

5.(a) Use Cramer's rule to solve the system of Equations

$3x_1 + x_2 - x_3 = -4$, $x_1 + x_2 - 2x_3 = -4$

$-x_1 + 2x_2 - x_3 = 1$

(b) Show that root of $x^2 + (mx+c)^2 = a^2$ will be equal if $c^2 = a^2(1+m^2)^2$

6.(a) Resolve $\dfrac{x^2+1}{x^3+1}$ into partial fractions.

(b) For what value of n, $\dfrac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the positive geometric mean between a and b.

7.(a) How many numbers greater than 1000,000 can be formed from the Digits 0, 2, 2, 2, 3 ,4 ,4

(b) Find the term independent of x in the expansion of $\left(x - \frac{2}{x}\right)^{10}$.

8.(a) Prove that $\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

(b) If $\tan\alpha = \frac{3}{4}, \cos\beta = \frac{5}{13}$ and neither the terminal side of the angle of measure α nor that of β is in the I quadrant, Find $\sin(\alpha+\beta)$

9.(a) Prove that in an equilateral triangle $r : R : r_1 = 1 : 2 : 3$

(b) Prove that $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$

Sargodha Board Group-I

Mathematics (Intermediate Part – I, Class 11th) Time : 30 Minutes
Session (2021) **Objective** Marks : 20

Note: You have four choices for each objective type question as A, B, C and D. the choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

1. G.M between 1 and 16 is/are
(a) 4 (b) -4 (c) ±4 (d) ±1/4

2. A.M between $\sqrt{2}$ and $3\sqrt{2}$ is
(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $\frac{4}{\sqrt{2}}$ (d) $\frac{\sqrt{2}}{2}$

3. Which of the following is an irrational number?
(a) $\sqrt{\frac{68}{17}}$ (b) $\frac{\sqrt{16}}{7}$ (c) $\frac{4}{\sqrt{2}}$ (d) $\sqrt{\frac{3}{27}}$

4. If a set S has 5 elements, Then number of improper subsetsare
(a) 1 (b) 15 (c) 31 (d) 32

5. The co-factor A_{22} of the matrix $\begin{bmatrix}1 & 2 & 4\\ -1 & 2 & 5\\ 0 & 1 & -1\end{bmatrix}$ is:
(a) 0 (b) -1 (c) 1 (d) 2

6. The matrix $\begin{bmatrix}1 & 2 & 0\\ 0 & 1 & 4\\ 0 & 0 & 6\end{bmatrix}$ is
(a) Diagonal (b) Scalar (c) Triangular (d) Singular

7. The quadratic equation $ax^2 + bx + c = 0$ becomes Linear equation if
(a) a = 0 (b) b = 0 (c) c = 0 (d) a = b

8. If w is complex roots or unity, Then value of $(3+\omega)(3+\omega^2) =$
(a) 6 (b) 7 (c) 9 (d) 13

9. If $\frac{7x+25}{(x+3)(x+4)} = \frac{A}{x+3} + \frac{B}{x+4}$, Then value of B is
(a) 3 (b) -3 (c) 4 (d) -4

10. $\sin\left(\sin^{-1}\left(\frac{1}{2}\right)\right) =$
(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

11. Solution of the equation cosx =-1 in $[0, 2\pi]$ is
(a) $\left\{0, \frac{\pi}{2}\right\}$ (b) $\{\pi\}$ (c) $\left\{\frac{-\pi}{2}, \frac{\pi}{2}\right\}$ (d) $\left\{\frac{\pi}{2}\right\}$

12. $(-1)^n, n \in N$ is a/an
(a) A.P (b) G.P (c) H.P (d) Series

13. A die is rolled, The probability of getting 3 or an Even number is
(a) $\frac{1}{12}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

14. Middle Term (S) of $(a+b)^{11}$ is/are
(a) 6th (b) 5th & 6th (c) 6th & 7th (d) 5th

15. $2\sin 45^\circ + \frac{1}{2}\operatorname{cosec} 45^\circ =$
(a) 1 (b) -1 (c) $\sqrt{2/3}$ (d) $\frac{3}{\sqrt{2}}$

16. If $\tan\theta > 0, \sin\theta < 0$, Then terminal arm of the angle θ will lie in quadrant.
(a) I (b) II (c) III (d) IV
17. If $\alpha = 30''$, then value of $\cot 3\alpha$ =
(a) 0 (b) 1 (c) 3 (d) ∞
18. The period of cosec 10x is
(a) $\frac{\pi}{10}$ (b) $\frac{2\pi}{5}$ (c) $\frac{4\pi}{5}$ (d) $\frac{\pi}{5}$
19. If α, β and γ are the angles of an ablique Triangle, then it must be true that
(a) $\alpha = 90''$ (b) $\beta = 90''$ (c) $\gamma = 90''$ (d) No angle is 90°
20. In any Triangle ABC, with usual notations, $\frac{a}{2\sin\alpha} =$
(a) Δ (b) r (c) 2R (d) R

Sargodha Board Group-I

MATHEMATICS PAPER-II **TIME ALLOWED : 2.30 Hours**
GROUP-II **SUBJECTIVE** **MAXIMUM MARKS:80**

NOTE: Write same question number and its part number on answer book, as given in the question paper.

SECTION-I

2. Attempt any eight parts.

(i) Prove that $\frac{-7}{12} - \frac{5}{18} = \frac{-21-10}{36}$
(ii) Simplify (5,-4) (-3,2)
(iii) Find the multiplicative Inverse of 1 – 2i.
(iv) Show that the statement $P \rightarrow (p \vee q)$ is tautology.
(v) Find the inverse of the relation $\{(x,y) | y^2 = 4ax, x \geq 0\}$
(vi) If a , b are elements of a group G. then show that $(ab)^{-1} = b^{-1}a^{-1}$
(vii) Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$
(viii) Without expansion show that $\begin{vmatrix} 6 & 7 & 8 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{vmatrix} = 0$
(ix) If $A = \begin{vmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 1 \end{vmatrix}$ if A_{12} and A_{22}
(x) Evaluate $(1+\omega-\omega^2)^8$
(xi) If α, β are the roots of the equation $3x^2 - 2x + 4 = 0$, find the value of $\alpha^3 + \beta^3$
(xii) Show that the roots of equation $px^2 - (p - q)x - q = 0$ will be rational.

3. Attempt any eight parts.

(i) Write only partial Fraction Form $\frac{x^2-2x+3}{x^4+x^2+1}$, without finding constants
(ii) Resolve $\frac{7x+25}{(x+3)(x+4)}$ into Partial Fraction.
(iii) If the nth term of an A.P is 3n - 1 Find the A.P
(iv) Find the 5th term of the G.P 3,6,12,.....
(v) Find the sum of an infinite geometric series $\frac{9}{4} + \frac{3}{2} + 1 + \frac{2}{3} + ...$

(vi) If the numbers $\frac{1}{k}, \frac{1}{2k+1}$ and $\frac{1}{4k-1}$ are in Harmonic Sequence, find k

(vii) Write (n + 2)(n + 1)(n) In the Factorial Form

(viii) How many 3-digit numbers can be Formed by using each one of the digits 2,3,5,7,9 only once?

(ix) If ${}^nC_8 = {}^nC_{12}$ find n.

(x) Prove the formula 1 + 5 + 9 + ... + (4n – 3) = n(2n – 1) For n = 1 , 2

(xi) Calculate $(0.97)^3$ by means of binomial theorem.

(xii) Expand $(4 - 3x)^{1/2}$ upto 4-terms

4. Attempt any nine parts.

(i) What is the circular measure of the angle between the hands of a watch at 4'O clock?

(ii) In which quadrant the terminal arms of the angle lie when sec $\theta < 0$ and sin $\theta < 0$

(iii) Prove that $\cos^2\theta - \sin^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

(iv) Find the value of tan (111 0)°

(v) Prove that $1 + \tan\alpha \tan(2\alpha) = \sec(2\alpha)$

(vi) Show that $\cot(\alpha-\beta) = \frac{\cot\alpha\cot\beta+1}{\cot\beta-\cot\alpha}$

(vii) Find the period of cos(2x)

(viii) Find the value of tan 19°30'

(ix) Find the area of the triangle ABC given three sides: a = 32.65 , b = 42.81 , c = 64.92

(x) Find the value of r if a = 34, b = 20 and c = 42

(xi) Without using table/calculator Prove that $\tan^{-1}(5/12) = \sin^{-1}(5/13)$

(xii) Find the value of θ satisfying $2\sin^2\theta - \sin\theta = 0$ $\theta \in [0, 2\pi]$

(xiii) Find the solution of $\operatorname{cosec}\theta = 2$

SECTION-II

Attempt any three.

5.(a) Show that $\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x0 \end{vmatrix} = (x+3)(x-1)^3$

(b) Show that the roots of $x^2 + (mx + c)^2 = a^2$ will be equal if $c^2 = a^2(1 + m^2)$

6.(a) Resolve into partial fraction $\frac{6x^3+5x^2-7}{2x^2-x-1}$

(b) The sum of 9 terms of an A.P is 171 and its eighth term is 31. Find the series.

7.(a) Prove that ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(b) Use mathematical induction to prove that the formula $1+\frac{1}{2}+\frac{1}{4}+.....+\frac{1}{2^{n-1}} = 2\left[1-\frac{1}{2^n}\right]$ is true for every positive integer n.

8.(a) Prove that $\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1-\sin^2\theta\cos^2\theta)$

(b) Prove that $\frac{\sin\theta+\sin3\theta+\sin5\theta+\sin7\theta}{\cos\theta+\cos3\theta+\cos5\theta+\cos7\theta} = \tan4\theta$

9.(a) Prove that $abc(\sin\alpha+\sin\beta+\sin\gamma) = 4\Delta\gamma$

(b) Prove that $\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{7}{25}\right) = \cos^{-1}\left(\frac{253}{325}\right)$

Faisalabad Board Group-1

(Intermediate Part – I, Class 11th)

Objective

Mathematics
Session (2021)

Time : 30 Minutes
Marks : 20

Note: You have four choices for each objective type question as A, B, C and D. the choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

1. A coin is tossed 5 times, then total number of outcomes n(S) = :
(a) 10 (b) 25 (c) 20 (d) 32

2. 2nd term in the expansion of $(1-x)^{-1}$ is:
(a) 1 (b) 2x (c) x (d) -x

3. $\sec\theta.\operatorname{cosec}\theta.\sin\theta.\cos\theta =$
(a) 1 (b) -1 (c) 0 (d)cannot be determined

4. In a right angled triangle, the side opposite to right angle is called:
(a) Base (b) Hypotenuse (c) Perpendicular(d) Altitude

5. If $\sin\alpha = \frac{2}{3}, \cos\alpha = \frac{3}{4}, then\ value\ of\ \sin 2\alpha =$
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{15}{144}$ (d) 1

6. Period of 2 + cos3x is:
(a) π (b) $\frac{3\pi}{2}$ (c) 2π (d) $\frac{2\pi}{3}$

7. In any triangle ABC , a = 4, b = 10, $\gamma = 30''$, then area of triangle $\gamma = 30''$ = :
(a) 5 sq.units (b) 10 sq.units (c) 20 sq.units (d) 40 sq.units

8. If a, b and c are the sides of a triangle ABC, then $\frac{c^2+b^2-a^2}{2bc}$ =:
(a) $\cos\alpha$ (b) $\cos\gamma$ (c) $\cos\beta$ (d) $\cos^2\alpha$

9. If $\sin^{-1}a = 0$, then value of a is:
(a) $\frac{\pi}{2}$ (b) π (c) 0 (d) 0 & π

10. The solution of the equation $2\sin x + \sqrt{3} = 0$ in 4th quadrant is:
(a) $\frac{\pi}{3}$ (b) $\frac{5\pi}{3}$ (c) $\frac{-\pi}{4}$ (d) $\frac{-\pi}{6}$ se roots

11. $(7,9)+(3,-5)=$:
(a) $(7,9)$ (b) $(3,-5)$ (c) $(10,4)$ (d) $(4,10)$

12. Which cannot be used as binary operations?
(a) Addition '+' (b) Division ÷ (c)Multiplication × (d)Square root $\sqrt{\ }$

13. If adjoint of a matrix $A = \begin{bmatrix} -1 & -2 \\ 3 & 4 \end{bmatrix}$, then matrix A is: : numbers.
(a) $\begin{bmatrix} -1 & -2 \\ 4 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -4 & 3 \\ -2 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 4 \\ -3 \end{bmatrix}$

14. Rank of the matrix $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ is: lues of
(a) 0 (b) 1 (c) 2 (d) 3

15. Which equation has roots 2 and -3?
(a) $x^2+x+6=0$ (b) $x^2+x-6=0$ (c) $x^2-x-6=0$ (d) x^2-x

16. If roots of the equation $x^2+px+q=0$ are additive inverse of each other, th is true?
(a) p = 0 (b) q = 1 (c) p = 1 (d) p =

17. $\frac{x^2+x-1}{Q(x)}$ will be an improper fraction, If:

(a) Degeree of Q(x) = 2 (b) Degeree of Q(x) > 2
(c) Degeree of Q(x) = 3 (d) Degeree of Q(x) ≠ 2

18. Sum of 5 A.Ms between 2 and 8 is:

(a) 10 (b) 40 (c) 25 (d) 50

19. If a and b are negative distinct real numbers, then with usual notations, which is correct?

(a) A > G (b) H < G (c) A < G (d) A > G >H

20. Which cannot be the term of a G.P.?

(a) -1 (b) 0 (c) 1 (d) 5

Faisalabad Board Group-I

MATHEMATICS PAPER-II **TIME ALLOWED : 2.30 Hours**
GROUP-II **SUBJECTIVE** **MAXIMUM MARKS:80**

NOTE: Write same question number and its part number on answer book, as given in the question paper.

SECTION-I

2. **Attempt any eight parts.**

(i) Does the set {1,-1} possess closure property with respect to addition and subtraction?

(ii) Find the difference and product of the complex number (8,9) and (5, -6)

(iii) Find the multiplicative inverse of $\left(\sqrt{2},-\sqrt{5}\right)$

(iv) If U = {1,2,3,4,5......,20} and A = {1,3,5,.......,19} verify $A\cup A'=U$

(v) Write the inverse and contrapositve of the conditional $\sim q \longrightarrow \sim p$

(vi) If a , b are elements of a group G then show that $(ab)^{-1}=b^{-1}a^{-1}$

vii) Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix}=\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

(ii) If A and B are square matrices of the same order, then explain why in general (x $+B)(A-B)\neq A^2-B^2$

Without expanasion verify that $\begin{vmatrix} \alpha & \beta+\gamma & 1 \\ \beta & \gamma+a & 1 \\ \gamma & \alpha+B & 1 \end{vmatrix}=0$

5.(a) Use the remainder theorem to find remainder when first polynomial is divided by econd polynomial $x^2+3x+7, x+1$

(b) nd the condition that one root of the equation $x^2+px+q=0$ is multiplicative ~erse of the other.

$c^2 = a^2$ (1 cuss the nature of roots of the equation $2x^2+5x-1=0$

empt any eight parts.

6.(a) Re olve $\frac{3x-11}{(x^2+1)(x+3)}$ into partial fractions without finding constants.

(b) Th

7.(a) Prc $\frac{6x^3+5x^2-7}{2x^2-x-1}$ into proper rational fraction.

(b) Use ge

$1+\frac{1}{2}+$ nd the indicated term of the sequence 1, -3, 5, -7, 9, -11........, a_8.

a_{n-3} = 2n - 5 find the nth term of the sequence.

8.(a) nd the nth term of the geometric sequence if $\frac{a_5}{a_3}=\frac{4}{9}$ and $a_2=\frac{4}{9}$

(b) Pr d A, G, H and verify that A > G > H (G>0) if a = 2 and b = 8 .

te n(n-1)(n-2)....... (n – r + 1) in factorial form.

9.(a) Pr d the value of n when $^nP_2=30$

e is rolled. What is the probability that the dots on the top are greater than 4.

(b) Prov binomial theorem expand $(a+2b)^5$

(xi) Expand $(1-x)^{\frac{1}{2}}$ up to 4 terms.

(xii) Using binomial theorem to find the values of

4. Attempt any nine parts.

(i) Find $l, when\, \theta = \pi\, radians,\ r = 6cm$

(ii) Verify $\sin^2\frac{\pi}{6}+\sin^2\frac{\pi}{3}+\sin^2\frac{\pi}{4}=2$

(iii) Prove that $\sec^2\theta-\operatorname{cosec}^2\theta=\tan^2\theta-\cot^2\theta$

(iv) Without using the table, find the value of sin(-300°)

(v) Prove that $\cos(\alpha+45^0)=\frac{1}{\sqrt{2}}(\cos\alpha-\sin\alpha)$

(vi) Prove that $\cot\alpha-\tan\alpha=2\cot 2\alpha$

(vii) Find the period of $\tan\frac{x}{7}$

(viii) Find the greatest angle of triangle ABC if a = 16, b = 20, c = 33

(ix) Find area of triangle ABC, given sides are a = 18, b = 24, c = 30

(x) Prove that $r_1 r_2 r_3 = rs^2$

(xi) Find the value of $\tan\left(\cos^{-1}\frac{\sqrt{3}}{2}\right)$

(xii) Find the solution of the equation $\sin x=\frac{-\sqrt{3}}{2}$ which lies in $[0,2\pi]$

(xiii) Solve $\cot\theta=\frac{1}{\sqrt{3}}$ where $\theta\in[0,2\pi]$

SECTION-II

5.(a) Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix}=(a+b+c)(a-b)(b-c)(c-a)$

(b) If α,β are the roots of equation $ax^2+bx+c=0$ then form the equation whose roots are $\frac{1}{\alpha^3},\frac{1}{\beta^3}$

6.(a) Resolve into partial fractions: $\frac{9}{(x+2)^2(x-1)}$

(b) If the H.M and A.M between two number are 4 and $\frac{9}{2}$ respectively, find the numbers.

7.(a) Prove that ${}^{n-1}C_r+{}^{n-1}C_{r-1}={}^nC_r$

(b) Determine the middle term in the expansion of $\left(\frac{1}{x}-\frac{x^2}{2}\right)^{12}$

8.(a) If $\cot\theta=\frac{5}{2}$ and the terminal are of the angle is in the I-quadrant, find the values of

(b) Prove that $\frac{2\sin\theta\sin 2\theta}{\cos\theta+\cos 3\theta}=\tan 2\theta\tan\theta$

9.(a) Show that $\frac{1}{r^2}+\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}=\frac{a^2+b^2+c^2}{\Delta^2}$

(b) Prove that $2\tan^{-1}\frac{2}{3}=\sin^{-1}\frac{12}{13}$

Gujranwala Board Group-I

Mathematics
Session (2021)

(Intermediate Part – I, Class 11th)
Objective

Time : 30 Minutes
Marks : 20

Note: You have four choices for each objective type question as A, B, C and D. the choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

1. For any two real numbers a and b, G^2 is equal to:

(a) AH (b) $\frac{H}{A}$ (c) $\frac{A}{H}$ (d) $\sqrt{AH}$

2. $\cos(-60^{\circ}) =$

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{-\sqrt{3}}{2}$

3. If $a_n = n + (-1)^n$, then a_{10} = :

(a) 10 (b) 11 (c) 9 (d) -11

4. $\frac{P(x)}{x^2+1}$ is proper fraction, if degree of the polynomial P(x) is:

(a) Equal to 2 (b) Greater than 2 (c) Not equal to 2 (d) Less than 2

5. Degree of a constant polynomial is:

(a) 1 (b) 0 (c) 2 (d) Arbitary

6. If one root of $x^2 + ax + 2 = 0$ is 2, then value of a is:

(a) -3 (b) 4 (c) 3 (d) -2

7. If $A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$, then cofactor of 6 is:

(a) 1 (b) -6 (c) -1 (d) 3

8. The matrix [7] is:

(a) Row matrix (b) Square matrix (c) Column matrix (d) All these

9. If $\sim p \rightarrow q$ is a conditional, then its converse is:

(a) $q \rightarrow \sim p$ (b) $\sim q \rightarrow p$ (c) $p \rightarrow \sim q$ (d) $\sim p \rightarrow \sim q$

10. If r is the radius and C is the circumference of a circle, then value of $\frac{C}{r}$ =:

(a) π (b) $\frac{\pi}{2}$ (c) 2π (d) $\frac{1}{2\pi}$

11. The solution of equation $\tan x = \frac{1}{\sqrt{3}}$ lies in quadrants.

(a) I & II (b) I & III (c) II & IV (d) I & IV

12. If $x = \sin^{-1}\frac{\sqrt{3}}{2}$, then value of x is:

(a) $\frac{-\pi}{2}$ (b) $\frac{-\pi}{3}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

13. With usual notations, $\frac{abc}{\Delta}$ =:

(a) 4R (b) r (c) R (d) rs

14. In any triangle ABC, if two sides and their included angle is given, then area of triangle is:

(a) $\Delta = \frac{1}{2}bc\sin\alpha$ (b) $\frac{1}{2}ab\sin\gamma$ (c) $\Delta = \frac{1}{2}ac\sin\beta$ (d) All these

15. Period of $2\operatorname{cosec}\frac{x}{4}$ is :

(a) $\frac{\pi}{2}$ (b) 4π (c) 2π (d) 8π

16. Value of $\sin 7\pi$ is equal to:

(a) 1 (b) ½ (c) -1 (d) 0

17. Angle $\frac{5\pi}{9}$ lies in quadrant.

(a) I (b) III (c) II (d) IV

18. If $l = 1.5cm, r = 2.5cm$, then value of θ is:

(a) 3.75 rad (b) $\frac{3}{5}rad$ (c) 0.60 rad (d) $\frac{5}{3}rad$

19. The 2nd term in the expansion of $(1-2x)^{\frac{1}{3}}$ is:

(a) $-\frac{2}{3}x$ (b) $\frac{2}{3}x$ (c) $\frac{4}{9}x^2$ (d) $\frac{3}{2}x$

20. The number of permutations of the word PANAMA are

(a) 10 (b) 60 (c) 20 (d) 120

Gujranwala Board Group-I

MATHEMATICS PAPER-II TIME ALLOWED : 2.30 Hours

GROUP-II **SUBJECTIVE** MAXIMUM MARKS:80

NOTE: Write same question number and its part number on answer book, as given in the question paper.

SECTION-I

2. Attempt any eight parts.

(i) Prove $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$ by rules of addition.

(ii) Factorize a^2+4b^2

(iii) Simplify $(2+\sqrt{-3})(3+\sqrt{-3})$

(iv) If U = If U= {1,2,3,..., 20} and A= {l,3,5,...,19} verify A ∪ A'=U

(v) Write inverse and contrapositive of the conditional ~ p → q

(vi) For A {1, 2, 3, 4), find the relation { (x, y) I x + y > 5) in A.

(vii) Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix}=\begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$

(viii) Without expansion show that $\begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{vmatrix}=0$

(ix) Find the inverse of matrix A = $\begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$

(x) Write second property of cube roots of unity without proof.

(xi) Find the remainder by using remainder theorem when first polynomial is divided by second polynomial $x^2 + 3x + 7$, $x + 1$

(xii) Show that the roots of the equation $(p + q) x^2 - px - q = 0$ will be rational.

3. Attempt any eight parts.

(i) Write $\frac{x^2+x-1}{(x+2)^3}$ in form of partial fractions without finding the constants.

(ii) Write $\frac{1}{(x+1)^2(x^2-1)}$ in form of partial fractions without finding the constants.

(iii) Write 1st four terms of the sequence $a_n = (-1)^n(2n-3)$

(iv) Find G.M. between -2 and 8

(v) Find the sum of infinite geometric series $2,\sqrt{2},1,...$

(vi) Find the 9th term of harmonic sequence $\frac{1}{3},\frac{1}{5},\frac{1}{7},...$

(vii) There are 5 green and 3 red balls in a box, one ball is taken out. Find the probability that the ball is green.

(viii) Write in factorial form (n + 2)(n + 1)n

(ix) How many signals can be given by 5 flags of different colours using 3 flags at a time.

(x) Calculate $(0.97)^3$ by means of binomial theorem.

(xi) Expand $(4-3x)^{1/2}$ upto three terms taking value of x such that (s.t) the expansion is valid.

(xii) Determine the middle term in the expansion of $\left(\frac{1}{x}-\frac{x^2}{2}\right)^{12}$

4. Attempt any nine parts.

(i) If $\sin\theta = -\frac{1}{\sqrt{2}}$ and the terminal is not in quadrant III, find the value of $\cos\theta$

(ii) Verify that $\sin^2\frac{\pi}{6}:\sin^2\frac{\pi}{4}:\sin^2\frac{\pi}{3}:\sin^2\frac{\pi}{2}=1:2:3:4$

(iii) Prove that $\sec^2 A+\operatorname{cosec}^2 A = \sec^2 A \cdot \operatorname{cosec}^2 A$, where $A \neq \frac{n\pi}{2}, n \in Z$

(iv) If α, β, γ are the angles of a triangle then prove that $\sin(\alpha+\beta)=\sin\gamma$

(v) Prove that $\cos(\alpha+45^\circ)=\frac{1}{\sqrt{2}}(\cos\alpha-\sin\alpha)$

(vi) Prove that $\frac{\sin A+\sin 2A}{1+\cos A+\cos 2A}=\tan A$ (vii) Find the period of $\tan\frac{x}{7}$

(viii) When the angle between the ground and the sun is 30°, flag pole casts a shadow of 40 in long. Find the height of the top of the flag.

(ix) Find the measure of the greatest angle, if sides of the triangle are 16, 20, 33

(x) Find the area of the triangle when b = 25.4, $\gamma = 36''41'$, $\alpha = 45''17'$

(xi) Prove that $\tan^{-1}\frac{5}{12}=\sin^{-1}\frac{5}{13}$

(xii) Find the general solution of the trigonometric equation sec x =-2

(xiii) Solve the trigonometric equation and write the solution in the interval $[0, 2\pi]$ when $2\sin^2\theta-\sin\theta=0$

SECTION-II

5.(a) Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix}$ = (a + b +c) (a –b) (b –c) (c - a)

(b) Show that root of $x^2+(mx+c)^2=a^2$ will be equal if $c^2=a^2(1+m^2)$

6.(a) Resolve $\frac{x^2+1}{x^3+1}$ into partial fractions.

(b) If $y=\frac{x}{2}+\frac{1}{4}x^2+\frac{1}{8}x^3+......$ and $0<x<2$, then prove that $x=\frac{2y}{1+y}$

7.(a) Two dice are thrown. What is the probability that the sum of the number of dots appearing on them is 4 or 6?

(b) If x is so small that its square and higher powers can be neglected then shot that $\frac{\sqrt{1+2x}}{\sqrt{1-x}}\approx 1+\frac{3x}{2}$

8.(a) Prove that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=\sec\theta-\tan\theta$ where θ is not an odd multiple of $\frac{\pi}{2}$

(b) Prove without using tables/ calculator that sin19° cos11° + sin71° sin11° = 1/2

9.(a) P and Q aretwopointsin1inewith If the distance between P and Q be 30m and the angles of elevation of the top of the tree at P and Q be 12° and 150 respectively, find the height of the tree.

(b) Show that $2\tan^{-1}\frac{2}{3}=\sin^{-1}\frac{12}{13}$

Lahore Board Group-I

Mathematics
Session (2021)

(Intermediate Part – I, Class 11th)
Objective

Time : 30 Minutes
Marks : 20

Note: You have four choices for each objective type question as A, B, C and D. the choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

1. If α,β are the roots of $x^2-px-p-c=0$, then $\alpha\beta=$
(a) $-p-c$ (b) $p+c$ (c) $p-c$ (d) $-p+c$

2. Sum of all the four roots of unity is
(a) 1 (b) 0 (c) -1 (d) 2

3. Partial fraction of $\frac{x^2+1}{(x+1)(x-1)}$ will be of the form.
(a) $\frac{A}{x-1}+\frac{B}{x+1}$ (b) $\frac{A}{x+1}+\frac{Bx+C}{x-1}$ (c) $\frac{Ax+B}{x^2-1}$ (d) $1+\frac{A}{x+1}+\frac{B}{x-1}$

4. The A.M between $\sqrt{2},3\sqrt{2}$ is
(a) $\sqrt{6}$ (b) $-2\sqrt{2}$ (c) $2\sqrt{2}$ (d) $-\sqrt{6}$

5. Common ratio of G.P $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ is:
(a) $\pm\sqrt{\frac{a}{c}}$ (b) $\pm\sqrt{\frac{c}{a}}$ (c) $\pm\sqrt{\frac{b}{c}}$ (d) $\pm\sqrt{\frac{c}{b}}$

6. H.M between 3 and 7 is:
(a) 5 (b) $\sqrt{21}$ (c) $\frac{21}{5}$ (d) $\frac{5}{21}$

7. If A and B are two independent events, then $P(A\cap B)=$
(a) P(A) + P(B) (b) P(A) - P(B)
(c) $P(A\cup B)$ (d) P(A). P(B)

8. The number of terms in the expansion of $(a+x)^n$ are:
(a) n (b) n+1 (c) n-1 (d) 2n

9. The value of $\tan\theta$ *for* $\theta=30^o$ *is*
(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

10. $\frac{5\pi}{6}$ *radian* $=$
(a) 150° (b) 130° (c) 120° (d) 60°

11. If $\sin\alpha=4/5, 0<\alpha<\pi/2$ *then* $\cos\alpha=$
(a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{4}{5}$ (d) $\frac{3}{5}$

12. π is the period of
(a) $\sec\theta$ (b) $\operatorname{cosec}\theta$ (c) $\cot\theta$ (d) $\sin 3\theta$

13. In any triangle ABC, with usual notation $\sqrt{\frac{s(s-c)}{ab}}=$
(a) $\cos\frac{\gamma}{2}$ (b) $\cos\frac{\alpha}{2}$ (c) $\cos\frac{\beta}{2}$ (d) $\sin\frac{\alpha}{2}$

14. Radius of e-circle opposite to vertex 'A' of $\triangle ABC$ is:
(a) $\frac{\Delta}{s-a}$ (b) $\frac{\Delta}{s-c}$ (c) $\frac{\Delta}{s}$ (d) $\frac{\Delta}{s-b}$

15. $2\tan^{-1}(A)=$
(a) $\tan^{-1}\left(\frac{A}{1-A^2}\right)$ (b) $\tan^{-1}\left(\frac{A}{1+A^2}\right)$ (c) $\tan^{-1}\left(\frac{2A}{1-A^2}\right)$ (d) $\tan^{-1}\left(\frac{2A}{1+A^2}\right)$

16. Reference angle of $\sin x = \frac{1}{2}$ Is:

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

17. Every non terminating, non recurring decimal represents:

(a) Rational number (b) Irrational number
(c) Natural number (d) Whole number

18. If A and B are any subsets of U, then A – B =

(a) $A \cup B^c$ (b) $(A \cup B)^c$ (c) $(A \cap B)^c$ (d) $A \cap B^c$

19. A square matrix A = [a_{ij}] is called upper triangle matrix if:

(a) $a_{ij} = 0 \; for \; i < j$ (b) $a_{ij} = 0 \; for \; i > j$
(c) $a_{ij} \neq 0 \; for \; i > j$ (d) $a_{ij} = k \; for \; i < j$

20. The trivial solution of system of homogenous linear equation in three variables is:

(a) (0,0,1) (b) (0,1,0) (c) (0,0,0) (d) (0,-1,0)

Lahore Board Group-I

MATHEMATICS PAPER-II
GROUP-II
TIME ALLOWED : 2.30 Hours
SUBJECTIVE
MAXIMUM MARKS:80

NOTE: Write same question number and its part number on answer book, as given in the question paper.

SECTION-I

2. Attempt any eight parts.

(i) Prove that $\frac{a}{b} = \frac{ka}{kb}, k \neq 0$

(ii) Simplify (5 , - 4) ÷ (- 3 , - 8) and write the answer as a complex number.

(iii) Find the real imaginary parts of $\left(\sqrt{3}+i\right)^3$

(iv) If B = { 1 , 2 , 3 }, then find the power set of B, i.e., P(B)

(v) Construct the truth table for the statement : ~ (p - q) $(p \wedge \sim q)$

(vi) For the set A = { 1 , 2 , 3 , 4 }, find a relation in A which satisfy {(x , y) I y+ x = 5 }

(vii) Find the matrix X , if 2X — 3A = B and $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$

(viii) Find A^{-1} if $A = \begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix}$

(ix) Without expansion, show that $\begin{vmatrix} \alpha & \beta+\gamma & 1 \\ \beta & \gamma+\alpha & 1 \\ \gamma & \alpha+\beta & 1 \end{vmatrix} = 0$

(x) Prove that sum of cube roots of unity is zero i.e., $1 + w + w^2 = 0$

(xi) Find the numerical value of k, when the polynomial $x^3 + kx^2 - 7x + 6$ has a remainder of - 4 when divided by x +2.

(xii) Show that the roots of equation $x^2 + (mx + c)^2 = a^2$ will be equal if $c^2 = a^2 (1 + m^2)$

3. Attempt any eight parts.

(i) Resolve $\frac{4x^2}{\left(x^2+1\right)^2\left(x-1\right)}$ into partial fractions without finding the constants.

(ii) Resolve $\frac{7x+25}{(x+3)(x+4)}$ into partial fractions without finding the constants.

(iii) Write the first four terms of the sequence, $a_n = (-1)^n n^2$

(iv) If $a_{n-3} = 2n - 5$, find nth term of the sequence.

(v) Insert two G.M's between 2 and 16.

(vi) Sum the infinite geometric series $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+----$

(vii) Find the value of n, when ${}^{11}P_n = 11.10.9$ (viii) Evaluate ${}^{12}C_3$

(ix) A die is rolled. What is the probability that the dots on the top are greater than 4?

(x) Check the truth of the statement $1+5+9+----+(4n-3) = n(2n-1)$ for n = 1,2

(xi) Calculate by means of binomial theorem (2.02)

(xii) If x is so small that its square and higher powers can be neglected, then show that $\frac{\sqrt{1+2x}}{\sqrt{1-x}} \approx 1+\frac{3}{2}x$

4. Attempt any nine parts.

(i) Convert 54°45′ into radians.

(ii) If $\cot\theta = \frac{15}{8}$ and the terminal arm of the angle is not in quadrant 1, find the value of cosec θ.

(iii) Verify $2\sin 45° + \frac{1}{2}\operatorname{cosec}45° = \frac{3}{\sqrt{2}}$ (iv)Prove that $\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}$

(v) Prove that $\tan(180° + \theta) = \tan\theta$

(vi) Express $2\sin 7\theta \sin 2\theta$ as sums or differences. (vii) Find the period of $\tan\frac{x}{7}$

(viii) A vertical pole is 8 m high and the length of its shadow is 6m. What is the angle of elevation of the sun at that moment?

(ix) Find area of the triangle ABC if a = 200 , b = 120 , γ = 150°

(x) Prove that $r\, r_1 r_2 r_3 = \Delta^2$

(xi) Find the value of $\sec\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$ (xii) Show that $r = (s-a)\tan\left(\frac{\alpha}{2}\right)$

(xiii) Find the solution of cosec $\theta = 2$ which lies in the interval $[0, 2\pi]$

SECTION-II

5.(a) Solve the system of linear equations by Cramer's rule.
$2x_1 - x_2 + x_3 = 8,\ x_1 + 2x_2 + 2x_3 = 6,\ x_1 - 2x_2 - x_3 = 1$

(b) If α, β are roots of equation $ax^2 + bx + c = 0$ from the equation whose roots are $\alpha+\frac{1}{\alpha}, \beta+\frac{1}{\beta}$

6.(a) Resolve $\frac{3x-11}{(x^2+1)(x+3)}$ into partial fractions

(b) If $S_n = n(2n-1)$ then find the series.

7.(a) Prove that ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(b) Use mathematical induction to prove $\binom{3}{3}+\binom{4}{3}+\binom{5}{3}+---\binom{n+2}{3} = \binom{n+3}{4}$ fo every positive integers n.

8.(a) Two cities A and B lies on the equator, such that their longitudes are 45° E and 25° W respectively. Find the distance between the two cities, taking the radius of the earth a 6400 kms.

(b) Prove that $\frac{\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta} = \tan 4\theta$

9.(a) Solve the triangle ABC, if a = 53, $\beta = 88°36'$, $\gamma = 31°54'$

(b) Prove that $\tan^{-1}\frac{1}{11} + \tan^{-1}\frac{5}{6} = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$

Bahawalpur Board Group-I

Mathematics
Session (2021)

(Intermediate Part – I, Class 11th)

Objective

Time : 30 Minutes
Marks : 20

Note: You have four choices for each objective type question as A, B, C and D. the choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

1. "0" is
(a) Irrational number (b) Positive integer
(c) Rational number (d) Negative integer

2. The set $\{x|x \in R \wedge x \neq x\}$ is
(a) Empty set (b) infinite set
(c) Singleton set (d) Binary set

3. Which of the following has no inverse?
(a) Identity matrix (b) Singular matrix
(c) Diagonal matrix (d) Non singular matrix

4. If order of the matrix A is $m \times n$ and order of B is $n \times P$ then order of AB is equal to
(a) $P \times m$ (b) $m \times m$ (c) $n \times n$ (d) $m \times P$

5. If 1, w, w^2 are cube roots of unity then $w + w^2 =$
(a) 1 (b) w (c) -1 (d) 0

6. The degree of the polynomial $ox^{15} + x^{14} + x^{12} + 5$ is
(a) 15 (b) 14 (c) 12 (d) 5

7. If degree of P(x) is less than degree of Q(x) then rational fraction $\frac{P(x)}{Q(x)}$ is called.
(a) Proper rational fraction (b) Improper rational fraction
(c) Common fraction (d) Rational number

8. Next of the sequence 7,9,12,16,... is.
(a) 20 (b) 21 (c) 22 (d) 23

9. A.M between $2+\sqrt{3}$ and $2-\sqrt{3}$ is:
(a) 4 (b) $\sqrt{3}$ (c) 2 (d) $2\sqrt{3}$

10. No term of a Harmonic sequence can be
(a) 1 (b) -1 (c) 2 (d) 0

11. Factorial of 0 i.e 0! is equal to
(a) 2 (b) 0 (c) Does not exists (d) 1

12. The number of terms in the binomial expansion $(a+x)^6$ are
(a) 7 (b) 6 (c) 5 (d) 4

13. 60^{th} part of a minute is called.
(a) Second (b) Minute (c) Degree (d) Hour

14. $\frac{1}{2}$ Rotation in clock wise direction equal to
(a) 180° (b) -180° (c) 90° (d) -90°

15. $\sin\left(\frac{\pi}{2}+\alpha\right)$ equals to
(a) $-\cos\alpha$ (b) $\sin\alpha$ (c) $\cos\alpha$ (d) $-\sin\alpha$

16. Period of Secant Function is
(a) π (b) 3π (c) 4π (d) 2π

17. If any triangle ABC with usual notations $\frac{b^2+c^2-a^2}{2bc}$ equal to
(a) $\cos\beta$ (b) $\cos\alpha$ (c) $\sin\beta$ (d) $\sin\alpha$

18. If the sides of a triangle are 18,24,30 then the value of S is
(a) 36 (b) 72 (c) 144 (d) 24

19. The function y = cosx is called principal cosine if
(a) $\frac{-\pi}{2} \le x \le \frac{\pi}{2}$ (b) $\frac{-\pi}{2} < x < \frac{\pi}{2}$ (c) $0 < x < \pi$ (d) $0 \le x \le \pi$

20. If $\sin x = \frac{-1}{\sqrt{2}}$ then the reference angle is
(a) $\frac{\pi}{3}$ (b) $\frac{-\pi}{4}$ (c) $\frac{-\pi}{3}$ (d) $\frac{\pi}{4}$

Bahawalpur Board Group-I

MATHEMATICS PAPER-II TIME ALLOWED : 2.30 Hours

GROUP-II **SUBJECTIVE** MAXIMUM MARKS:80

NOTE: Write same question number and its part number on answer book, as given in the question paper.

SECTION-I

2. Attempt any eight parts.

(i) Prove that $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$

(ii) Simplify(2,6) (3,7)

(iii) Factorize $a^2 + 4b^2$

(iv) Verify the commutative property of union if A = {1 ,2,3,4, 5} ; B {4,6, 8, 10}

(v) Write two proper subsets of {a , b , c}

(vi) Find the inverse of the relation {(1,3), (2,5), (3,7), (4,9), (5,11)}

(vii) Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix}=\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

(viii) If $A=\begin{bmatrix} 2 & -1 & 3 & 0 \\ 1 & 0 & 4 & -2 \\ -3 & 5 & 2 & -1 \end{bmatrix}$ then find AA^t

(ix) If $A=\begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix}$ show that $A + (A)^t$ is hermitian.

(x) Solve $x^2+7x+12=0$ by factorization.

(xi) Show that $x^3-y^3=(x-y)(x-wy)(x-w^2y)$

(xii) Show that the roots of the equation $(P + q)x^2 - Px - q = 0$ will be rational.

3. Attempt any eight parts.

(i) Resolve into partial fraction $\frac{x^2+x-1}{(x+2)^3}$ without find in values of unknown constants.

(ii) Resolve $\frac{7x+25}{(x+3)(x+4)}$ into partial fraction.

(iii) Find the next two terms of 1,3,7,15,31...

(iv) Find the Arithmetic Mean (A.M) between x − 3 and x + 5

(v) Find the sum of Geometric progression $2,\sqrt{2},1,\ldots$

(vi) Find the 12th term of $\frac{1}{3},\frac{2}{9},\frac{1}{6},\ldots$

(vii) Find the value of n, when ${}^{11}P_n = 11.10.9$

(viii) What is the probability that a slip of numbers 1 , 2, 3 . 10 ?

(ix) A dic is thrown twice, what is the probability that the sum of the numbers of dots shown 3 or 11

(x) Evaluate $\sqrt[3]{30}$ correct to the three decimal.

(xi) Use mathematical induction to prove that 1 + 5 + 9 + ... + (4n − 3) = n(2n − 1) is ture for n = 1 and n= 2

(xii) Determine the middle term of the expansion $\left(\frac{1}{x}-\frac{x^2}{2}\right)^{12}$

4. Attempt any nine parts.

(i) Find the value of $\sin\theta$ and $\cos\theta$ if $\theta = \frac{-9\pi}{2}$

(ii) Prove that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$ where θ is not an odd multiple of $\frac{\pi}{2}$

(iii) Convert 54° 45' into radian.

(iv) Prove that $\sin\left(\frac{\pi}{4}-\theta\right)\sin\left(\frac{\pi}{4}+\theta\right) = \frac{1}{2}\cos 2\theta$

(v) Prove that $\cot\alpha - \tan\alpha = 2\cot 2\alpha$

(vi) Without using calculator, prove that cos 330° sin 600° + cos 120° sin 150° = -1

(vii) Find the period of cos 2 x

(viii) The area of triangle is 121.34 . If α = 32° 15'. β = 65°37' ,then find c and angle γ.

(ix) Prove that $\frac{1}{r^2}+\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2} = \frac{a^2+b^2+c^2}{\Delta^2}$

(x) Solve the right triangle ABC in which γ = 90° and α = 62° 40', b = 796

(xi) Show that $\sin^{-1}(-x) = -\sin^{-1}x$

(xii) Solve the equation $\cot\theta = \frac{-1}{\sqrt{3}}, \theta \in [0, 2\pi]$

(xiii) Find the value of θ in $[0, 2\pi]$ satisfying the equation $2\sin\theta + \cos^2\theta - 1 = 0$

SECTION-II

Attempt any three.

5.(a) Find the rank of the matrix $\begin{bmatrix} 1 & -4 & -7 \\ 2 & -5 & 1 \\ 1 & -2 & 3 \\ 3 & -7 & 4 \end{bmatrix}$

(b) Prove that $\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$ will have equal roots if

$c^2 = a^2m^2 + b^2, a \neq 0, b \neq 0$

6.(a) Resolve into partial fractions $\frac{9}{(x+2)^2(x-1)}$

(b) Find four number in A.P whose sum is 32 and sum of whose squares is 276.

7.(a) A natural number is chosen out of the first fifty natural numbers. What is the probability that the chosen number is a multiple of 3 or of 5.

(b) Use mathematical induction to prove that

$1^3 + 3^3 + 5^3 + \ldots + (2n-1)^3 = n^2(2n^2 - 1)$

8.(a) Find the values of the trigonometric functions of the angle $\theta = \frac{-17}{3}\pi$

(b) Prove without using calculator $\sin 19^o \cos 11^o + \sin 71^o \sin 11^o = \frac{1}{2}$

9.(a) Prove that $\tan^{-1}\frac{1}{11} + \tan^{-1}\frac{5}{6} = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$

(b) Solve the triangle ABC in which $a = \sqrt{3}-1, b = \sqrt{3}+1$ and $\gamma = 60^o$

Rawalpindi Board Group-I

Mathematics (Intermediate Part – I, Class 11th) Time : 30 Minutes
Session (2021) **Objective** Marks : 20

Note: You have four choices for each objective type question as A, B, C and D. the choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

1. If $a_n = (-1)^n(2n-3)$ Then a_5 =
(a) 7 (b) -7 (c) 13 (d) -13

2. Multiplicative inverse of –i is:
(a) I (b) -I (c) 1 (d) -1

3. Tabular form of $\{x \mid x \in E \wedge 4 < x < 6\}$ is
(a) { } (b) {4} (c) {6} (d) {4,6}

4. If $A = \begin{bmatrix} 1 & 2 & -2 \\ 0 & 0 & 6 \\ 6 & 7 & 4 \end{bmatrix}$ then A_{33} =
(a) -1 (b) 1 (c) -2 (d) 0

5. A matrix of order $1 \times n$ is called
(a) Row matrix (b) Column matrix (c) Diagonal matrix (d) Null matrix

6. If one root of equation $x^2 + px + q = 0$ is additive inverse of other, then
(a) p = -1 (b) p = 0 (c) q = 1 (d) q = 0

7. If ω is cube root of unity, then $\omega + \omega^2 = 0$
(a) 0 (b) -1 (c) 1 (d) $\frac{1}{\omega}$

8. Partial fraction of $\frac{1}{(x+1)(x^2-1)}$ will be of the form.
(a) $\frac{A}{x+1} + \frac{Bx+c}{x^2-1}$ (b) $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1}$
(c) $\frac{A}{x+1} + \frac{B}{x-1} + \frac{Cx+D}{(x+1)^2}$ (d) $\frac{A}{x+1} + \frac{Bx+C}{x^2+1}$

9. Arithmetic mean between a and b is
(a) $\frac{a-b}{2}$ (b) $\pm\sqrt{ab}$ (c) $\frac{2ab}{a+b}$ (d) $\frac{a+b}{2}$

10. If Sinx = Cosx then x =
(a) 45° (b) 30° (c) 0° (d) 60°

11. G.M between 2i and 8i equals
(a) ±4 (b) $5i$ (c) -4 (d) $\pm 4i$

12. For independent events $P(A \cap B)$ =
(a) P(A) + P(B) (b) P(A) - P(B) (c) P(A) . P(B) (d) $\frac{P(A)}{P(B)}$

13. Expansion of $(1-2x)^{1/3}$ is valid, if
(a) $|x| < 1$ (b) $|x| < 1/3$ (c) $|x| < 2$ (d) $|x| < \frac{1}{2}$

14. $\cot^2\theta - \operatorname{cosec}^2\theta$ =
(a) 1 (b) -1 (c) 0 (d) 2

15. $\cos(-60^\circ)$ =
(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{-\sqrt{3}}{2}$

16. $\cos 2\alpha$ =
(a) $2\sin^2\alpha - 1$ (b) $2\cos^2\alpha - 1$ (c) $2\cos\frac{\alpha}{2}\sin\frac{\alpha}{2}$ (d) $1-2\cos^2\alpha$

17. Period of Cot 8 x is

(a) 8π (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{4}$ (d) π

18. $\cot\frac{\alpha}{2} =$

(a) $\sqrt{\frac{s(s-c)}{(s-b)(s-a)}}$ (b) $\sqrt{\frac{s(s-a)}{(s-b)(s-c)}}$ (c) $\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ (d) $\sqrt{\frac{s(s-a)}{(s-b)(s-c)}}$

19. In any Triangle ABC, with usual notation $\frac{b-c}{b+c} =$

(a) $\frac{\tan\frac{\beta-\gamma}{2}}{\tan\frac{\beta+\gamma}{2}}$ (b) $\frac{\tan\frac{\beta+\gamma}{2}}{\tan\frac{\beta-\gamma}{2}}$ (c) $\frac{\tan\frac{\alpha-\gamma}{2}}{\tan\frac{\alpha+\gamma}{2}}$ (d) $\frac{\tan\frac{\alpha+\beta}{2}}{\tan\left(\frac{\alpha-\beta}{2}\right)}$

20. Value of $\sec\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)$

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{2}}$

Rawalpindi Board Group-I

MATHEMATICS PAPER-II **TIME ALLOWED : 2.30 Hours**
GROUP-II **SUBJECTIVE** **MAXIMUM MARKS:80**

NOTE: Write same question number and its part number on answer book, as given in the question paper.

SECTION-I

2. Attempt any eight parts.

(i) Find the multiplicative inverse of(-4, 7)

(ii) Show that $\forall z_1, z_2 \in C, \overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

(iii) Find the difference of the complex numbers (8 , 9) and (5 , -6)

(iv) Show that the statement $(p \wedge q) \rightarrow p$ is a tautology

(v) If A = {a, {b,c}} then find P(A).

(vi) Write the set builder notation of the set, {0, ± 1, ± 2,... ± 1000}

(vii) Find the matrix X if: $X\begin{bmatrix}5 & 2\\ -2 & 1\end{bmatrix} = \begin{bmatrix}-1 & 5\\ 12 & 3\end{bmatrix}$

(viii) Show that $\begin{vmatrix}a+l & a & a\\ a & a+l & a\\ a & a & a+l\end{vmatrix} = l^2(3a+l)$

(ix) If $A = \begin{bmatrix}4 & \lambda & 3\\ 7 & 3 & 6\\ 2 & 3 & 1\end{bmatrix}$ is singular. Find the value of λ

(x) Evaluate $\left(1+\omega-\omega^2\right)^8$

(xi) Find the roots of the equation: $16x^2 + 8x + 1 =$ by using Quadratic formula.

(xii) By using remainder theorem, fmd the remainder when the polynomial $x^2 + 3x + 7$ is divided by x+1

3. Attempt any eight parts.

(i) Resolve into Partial Fractions, $x^2 - 1$

(ii) Write into Partial fractions without finding the constants $\frac{9}{(x+2)^2(x-1)}$

(iii) Find the indicated term of the following sequence 1, -3, 5, -7, 9, - 11,...,a_8.

(iv) If the nth term of the A.P is 3n + 1 , find arithmetic progression.

(v) Find the 12th term of the geometric sequence 1 + i ,2 i ,-2+2 i ----

(vi) If the numbers $\frac{1}{k}, \frac{1}{2k+1}$ and $\frac{1}{4k-1}$ are in harmonic sequence, find k.

(vii) Evaluate $^{16}P_4$

(viii) In how many ways can a necklace of 8 beads of different colours be made?

(ix) Find the value of n, when $^nC_5 = {^nC_4}$

(x) Calculate by means of binomial theorem $(0.97)^3$

(xi) Expand up to 3 terms $(1-x)^{1/3}$

(xii) If x is so small that its square and higher powers be neglected, then show that $\frac{\sqrt{4+x}}{(1-x)^3} \approx 2 + \frac{25}{4}x$

4. Attempt any nine parts.

(i) Convert 54° 45′ into radians. (ii) Verify $\sin^2\left(\frac{\pi}{6}\right) + \sin^2\left(\frac{\pi}{3}\right) + \tan^2\left(\frac{\pi}{4}\right) = 2$.

(iii) Prove that $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta \,\forall\, \theta \in R$

(iv) Without using tables write down the value of cos 315°

(v) Prove that tan(45° + A)tan (45° - A) = 1

(vi) Prove that $\frac{\sin A + \sin 2A}{1 + \cos A + \cos 2A} = \tan A$ (vii) Find the period of $3\cos\left(\frac{x}{5}\right)$

(viii) Find the value of Cot 89° 9′.

(ix) Find the area of △ ABC having a = 200, b = 120, $\gamma = 150°$

(x) In △ ABC if a = 13, b = 14, c = 15 find R.

(xi) Show that $\sin^{-1}(-x) = -\sin^{-1}(x)$ (xii) Solve the equation $\sin x = \frac{1}{2}$

(xiii) Find the solution of $\sin x = -\frac{\sqrt{3}}{2}$ which lie in $[0, 2\pi]$

SECTION-II

Attempt any three.

5.(a) Use Cramer's rule to solve the system of Equations

$3x_1 + x_2 - x_3 = -4$, $x_1 + x_2 - 2x_3 = -4$

$-x_1 + 2x_2 - x_3 = 1$

(b) Use synthetic division to find the values of p and q if x+1 and x-2 are the factors of the polynomial $x^3 + px^2 + qx + 6$

6.(a) Resolve into Partial fractions $\frac{9x-7}{(x^2+1)(x+3)}$

(b) If the (positive) Geometric Mean and Harmonic Mean between two numbers are 4 and $\frac{16}{5}$, find the numbers.

7.(a) Prove that $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$ (b) Find 6th term in the expansion of $\left(x^2 - \frac{3}{2x}\right)^{10}$

8.(a) If $\sin\theta = -\frac{1}{\sqrt{2}}$ and the teina1 a of angle is not in quad. III Find the values of remaining trigonometric functions.

(b) Prove that $\frac{2\sin\theta\sin 2\theta}{\cos\theta + \cos 3\theta} = \tan 2\theta \tan\theta$

9.(a) Prove that $r = 4R\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}$ (b) Prove that $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$

Answers (Sahiwal Board)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
B	C	A	B	A	C	D	C	D	B	B	D	A	B	C	B	B	C	D	A

Answers (Multan Board)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
C	A	B	D	A	D	A	B	C	C	A	A	B	D	A	D	A	C	A	C

Answers (D.G. Khan Board)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
D	B	C	C	C	D	C	B	C	B	D	A	C	D	B	A	B	A	C	D

Answers (Sargodha Board)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
C	B	C	C	B	C	A	B	A	A	B	A	D	C	D	C	A	D	D	D

Answers (Faisalabad Board)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
D	C	A	B	D	D	B	A	D	B	C	D	D	B	B	A	A	C	C	B

Answers (Gujranwala Board)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	A	B	D	B	A	C	D	A	C	B	C	A	D	D	D	C	C	A	D

Answers (Lahore Board)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	B	D	C	A	C	D	B	B	A	D	C	A	A	C	B	B	D	B	C

Answers (Bahawalpur Board)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
C	A	B	D	C	B	A	B	C	D	D	A	A	A	C	D	B	A	D	D

Answers (Rawalpindi Board)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
B	A	A	D	A	B	B	b	D	A	d	C	D	B	A	B	b	B	A	B

★★★★★★★★★